(ART)iculation Theory

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Abstract: Articulation theory is the study of how relations of subordination and dominance emerge between individuals across the frameworks of norms and institutions that organize their states of affairs. As such, we must formalize this development in a dynamic way. Below, I utilize vector notation to illustrate interactions between the frameworks individuals reference when organizing their experience, indicating how exchanges between two or more levels of society and/or individual (events, persons, institutions) constitute the state of affairs under consideration. Afterwards, I develop a method to apply this framework to analyze current states of affairs. In so doing, I propose an analysis of what are, by definition, dynamic, complex, adaptive and paradigmatically constructed phenomena.

Keywords: Articulation theory, Black studies, Cultural studies, dominance, methodology, Stuart Hall, subordination.

1. Introduction

Articulation theory studies how relations of subordination and dominance in socio-cultural and political affairs emerge and how these relations are reproduced across states of affairs. Its formalization was called for by Stuart Hall at a UNESCO conference in 1980. Since then, there have been attempts at formalization with some success – see Peterson II (2019; 2022). The current effort starts from those efforts and formalizes the next step, those models' applicability. The articulation of these relationships becomes a physical inquiry, treating those emergent relations and, thereby, the conditions themselves, as objects of study rather than merely documenting their consequences. Previously, it was of the utmost importance to treat these formations as processes, as vectors. Below we'll see how passing a vector describing a particular relationship between its components through a set of conditions formalized as a matrix allows us to test theories of state formation and, possibly, ascertain opportunities for state change. Matrix notation allows us to map how these relationships emerge and evolve without having to reduce them to a singular instance. Power can be described by relations between obtainable positions within a matrix, below a network of norms and institutions providing the infrastructure organizing what form life can take. The matrices organizing our states of affairs, then, are encoded and then transmitted across contexts (Scott 2022). How those relationships are reproduced in future contexts makes for a dynamic Articulation theory.

2. What is Articulation Theory

Articulation theory studies social formations and how conceptual frameworks structure personal, political, technological, and economic processes. Articulation – as in an assemblage, not an expression – provides a model for how relationships of subordination and dominance emerge between individuals and institutions. Stemming from the sociologist Stuart Hall's social communication model, cultural study analyzes the infrastructure of the networks of norms and institutions organizing our affairs, allowing or disallowing certain choices to individuals. In physics, a field frames a set of possible actions. If there are n-phases or operators to an event, then there are n-factorial ($n!$) possible ways to move from the initial to the resulting state. As such, the path over the sum of possible histories providing the probability of getting to what we've observed is $1/\sqrt{n!}$. The cultural studies interpretation of this has been introduced by Black studies scholar Saidiya Hartman as fabulation – explored below. Thus, the frame organizing what options are available to operators in a field is given by the infrastructure denoted by the converse of the shape taken up by what's observed in that field.

Classically, infrastructure is defined as the organizational structures that allow for the operation of a society. Articulation focuses on what's transmitted through these infrastructures. Choices transmit one's position in the network as what's identified by others. That identity references the framework that allowed for that selection. Thus, what is tacitly transmitted as the content of that identity, according to social theorists, is a representation of the frame that guides a decoding process whereby one relates, understands or identifies, another's position in that network relative to their own. Different structures operating across the same field means that different resources come available at different regions leading to asymmetrical relationships between them. The unevenness of that topology means that some regions obtain a subordinate position to others.

Hall called for the development of a model rather than an archive of articulation's effects. Models encode a theory abstracted from a particular state of affairs and sets principles that relate objects relevant in the model to subsequent observations. A model is 'good' insofar as it can describe a large class of observations from finite resources, making sense of relations between concepts and experience insofar as others can take up that model, apply it to similar conditions, and obtain comparable results. A model may be good insofar as its power to reproduce the relationships it encodes, but a model of models may determine the former's output as bad in relation to other models. Hence, the formalization of power relations explored below.

We'll utilize Hall's encoding/decoding model of social communication. An individual selects features from their environment and encodes a relationship between them, indexing a framework organizing a worldview that orients that individual to that environment in a particular way. The frame is used to organize or decode subsequent experiences such that the objects revealed by that worldview are objects whose features are related in the way indexed by that framework. The choices available to an individual given that frame imply a relationship to that environment such that, given a selection, that choice is identified within another's frame as representative of the frame employed by that individual. When frames are shared between individuals, their experience is organized such that the same relationship between features of the world are revealed, relating different objects under similar frameworks. This connection indexes an emergent norm between individuals. When a norm is encoded and shared, they institute worldviews. These worldviews can be shared by a group or held within an institution.

Institutions organize the output of individuals as well as groups while determining what's available to them, articulating their position with respect to others within that network. Institutions determine what types of relationships between identities are possible by structuring what experience is available to individuals using those resources to construct or update the frames they employ to make sense of their experience. This indirectly influences what and how they interact with other individuals and/or norms or institutions. Culture becomes about the object-relations through the infrastructure that determines what sort of expressions can emerge given those conditions, the set of those expressions becoming what is seen as the culture of that state from the vantage point of another when fed back into that system. What travels through these infrastructures and how that output, once fed back into that network, becomes significant between the individuals navigating that infrastructure with the means available to them is key.

An example is La-Toya Scott's (2022) work on social media networks and their material effects on socio-political organizing. Certain 'handles' and/or 'tags' encoding a range of messages online (e.g. Twitter and Instagram) allowing those able to decode those tags in terms relevant to their own situation to traverse that network in different ways. Scott's analysis of these 'hush harbors' – places where Blacks in the antebellum era of the U.S. were able to gather in plain sight, sharing information to organize survival and escape efforts – shows how the infrastructure of a particular network can be organized differently, leading to alternative output from the same domain. Here, digital hush harbors organized a domain of information and sharing practices that materialized in various locations, with the resources available, in ways relevant to the local situation. Object of relations between sets of practices concretized in Black Lives Matter efforts against legalized as well as extra-judicial policing. Non-local connections across the digital network mirrored the relations between seemingly unconnected but situationally similar geographies and contexts in the real world. Looking at the various territories distinguished in the mapping of discourse around particular topics mirrors which positions and locations were subordinated, the value of their output overdetermined, by others and how that relates to the positions and locations in the world from which that information is being transmitted, filtered through the network, and received elsewhere. The ability to put different things in those locations to use in similar ways, shows what appears superficially different can be related by the practices that put those things to use.

Articulation theory studies the relationship between the institutions that organize possible relations between individuals and groups in such a way that the norms upon which these institutions are built can reproduce themselves in context-relevant ways in the future. Once this network is revealed, the study of what identities are able to emerge and which are disallowed when instituted frameworks compete comes forward. Our model allows us to treat practices objectively, as functions indexing relations between input and output. Individuals become operators in the fields organized along these axes, explaining how culture shows up the way it does given the environment in which those practices operate. This allows us to see non-local connections via practices – i.e. functional equivalences – rather than output alone, (i.e. the same cultural product may appear different given the conditions in which it operates) as well as practices implemented to maintain the same surface level appearance regardless of environmental resources.

Below, we will first set out our goals for developing our articulatory model as outlined above. Then we will review a general background of Articulation to provide motivations for our study. We will go on to provide some tools and detail a method of application to bring the program we wish to elucidate into positive relief. Finally, we'll formalize the application of our model and apply it to two case studies to determine the benefit of these improvements.

2.1 Motivations

We wish to formalize the concept of overdetermination, i.e. how a position made available to certain agents within a particular state of affairs leads to the same value attributed to their output regardless of input. As such, the matrix organizing those affairs is reproduced over time. To do so, we must develop tools and methods of application that can be shared and whose output can be compared so as to test to those method's utility. This is important for we wish to retain Articulation theory as a study of dynamic entities.

We'll develop a model utilizing matrix notation as a convenient way to graph coordinate systems and describe a network of possible interactions between positions. Matrices are convenient for constructing models of world affairs, mapping relations between the constituents of the world of concern given the relationships between the coordinates we assign observables in our model. Relations between norms and institutions give way to a network through which the tacit transfer of worldviews are made by choices amongst those available with respect to one's position therein. As worldviews are encoded by frameworks organizing one's experience, a choice is evidence of one's access to that framework or that that framework has been transmitted from a previous context and applied to the current set of conditions – what Helena Miton and Simon DeDeo call 'tacit knowledge' of cultural practices, ways of traversing the terrain organized by said infrastructure. A network conception of the organizational structures required for the operation of society connects our conception of culture as infrastructure and the analysis of how relations of subordination and dominance emerge through studying the flow of resources as set out by Stuart Hall.

Our model is a formal way of mapping so as to interpret what was laid out above. Matrix notation helps to represent sets of relations indexed by a particular framework or worldview and see if what is transmitted through that matrix is significant with respect to that frame. Vector notation licenses the study of the formation of these relations as processes without reducing them to their output. This encapsulates the assumption that the iterative projection of frames across experience marks the objects they capture with a magnitude, i.e. significance, and that iteration implies a direction, their use evolves over time.

Below, a 3×2 matrix maps the interaction between the two 'state-descriptions' we wish to explore on its rows. A field composed of n=2 operators in n-states is considered by way of a norm that gauges the possible outcomes of their interaction which is set to $1/\sqrt{2}$ as the function characterizing that system. Each row is made up of a (w)orldview, (n)ame, and a (r)elation component. The w and n components are the basis vectors, here the axes of the encoded frame. A relation r between the evidence, the object identified or named within that frame, and that environment can be measured. The r component is the relationship between n in w being tested. If that relationship changes at most by a scalar factor across frames, then the r component represents a characteristic value that can be associated with the state of affairs framed by w and n, called an eigenvalue. The Articulation theory we propose constructs framings of an environment abstracted from a source or archive and tests our characteristic value at different scales to ascertain if our description of that state of affairs obtains from the objects that are revealed by our framing. The eigenvalue tests the extent to which the instituted/encoded relations between the objects captured by that framework are reproduced, i.e. overdetermined.

Our model is not meant to be predictive but explanatory in that we construct conditions from archival records in which we test if our theorized object relation is valid within that framework. When that framework is shared, and similar results arise, then an object-wise relationship between those states of affairs exhibits either the overdetermination of one's capacity to navigate that state or that that alternative interpretation is justified by our solution. Overdetermination is when an operator at one node of our matrix forecloses options at another by reproducing a prior set of relations between constituents regardless of what's available in this iteration. If similar but offering a different insight, we can update our model along the w and n basis. If refuted, then that alternative can be explored.

Overall, uptake of a general model of articulation that can be updated as it acquires evidence with each application seeks to avoid essentialism regarding the entities, the conditions, being analyzed. This will be shown as resulting from treating a method as the solution itself, universalizing singular outcomes over all historical and current analyses.

2.2 Concerns

In 1980, Stuart Hall made a call for the formalization of Articulation theory (ART-theory) while identifying a tendency to rely on strict historicism without showing, in the work, the model being used to abstract features from the record to explain social phenomena. In studying social-political formations one encodes a relation between observable features and projects that encoding in such a way that when others decode the outcomes of that model in terms relevant to them and apply it to the same record, an equivalence may be found between interpretations that lends credence to as well as aids in scrutinizing our results (Hall 1973). Hall thought this crucial as frames' results are always decoded in terms relevant to the receiver. Limits regarding the translation of output across applications means that we have a mechanism that doesn't rely on stipulation, but with which we determine what's considered significant in that field of study relative to frame. By retreating from formalizing methods that can be shared and utilized by others to test the extent to which they apply, knowledge production faces a crisis (Spillers 1994). Certain frames are valorized over others without scrutiny. A static historicity – in the Althusserian sense – falls prey to the universalist trap its authors try to avoid (Spillers 1994; Hall 1980; Wynter 2005). To extract and abstract a narrative from the archive at various points, generalizing it as the sole explanation for every situation, circumstance, and interpretation, leads to an infinite regress of binary identities relative to one's affiliated position that's based upon not being the other, halting study and limiting alternative modes of knowledge production (Wynter 2005).

Developing models means that we can test and find the limits of a frame's application and, if able to be reduplicated by others, those treatments can be compared and updated. This intersubjective basis is required for science (Hempel 1952). It also has the benefit of allowing us to study the object of ART-theory, power.

Relations of power are correlated with institutions of knowledge imposed upon a state which in turn implies an instituted power relation between positions obtainable therein (Foucault 1975). These frames can be considered by way of a grid or matrix that encodes a particular relationship to the things in the contexts in which it is applied (Foucault 1966). This conception keys us into the fact that it is not the things themselves but the relationships they obtain that indicates the imposition of this matrix. Different things organized particular ways indicate a similar system at work. The same category can be made of different things, the same thing obtain different categories, different things across contexts obtaining the same category being functionally equivalent relative to the system employed. Power relations were of interest to Hall for the formation of these matrices is a process that abstracts from experience and encodes that relationship to organize subsequent contexts, so that when decoded in that context's successors, organizing the following experience in similar ways, it can be said that we know something about that context (Hall 1973). The capacity to reproduce knowledge over a context's constituents and, at times, despite them, is where power comes into play.

Knowledge over contexts and the study of systems purporting knowledge over other contexts in excess of the knowledge employed within them constitutes the basis of ART-theoretical study; both its connections to studies of Race and racialization, as well as to appropriation and colonial legacy. The formalization of the emergence, development, and extension of these matrices brings in efforts to study these conditions as objects themselves and, therefore, the possibility of rearranging that network of socio-cultural and political positions, i.e. the infrastructure (re)producing it. The rearranging of those nodes, creating alternatives arrangements from apparent resources, means that the possible arrangements of those constituents exceed. The constitutive objects of a given domain. Matrices of power can be re-articulated (Quijano 2000). The limits to which this can be done is not a failing of formalization, but a guide back towards the capacity to form the matrices organizing our experience in the first place.

2.3 Formalization

By modeling relationship formation, we can prepare tests of alternatives through a particular state of affairs or field. This field can be represented as a matrix that encodes relationships between various nodes representing some position obtainable in a socio-cultural or political arrangement. The matrix codifying this network organizes the affairs over which it is projected. Consequently, the labels for these nodes are reified in our experience as we refer to them to explain state affairs. Above we spoke of functional equivalences between different objects obtaining similar positions across different states organized by the same matrix. Here we will make sense of this by showing that if the relationships between social output holds across contexts, then we that output is structured by tags that are strictly determined and inheritable (Bright et al 2022).

To show this, we'll develop the concept of a power score that can be compared across contexts, "a single number that summarizes group consensus on the basis of individual interactions." (DeDeo 2016, 7, 9-10) These interactions are the relationships obtained and obtainable within a state. They're noted by dyadic interactions of an n-element list represented by an $n \times n$ matrix. Previous ART-theoretical models have shown how one compresses or encodes such a matrix by virtue of its determinant, projecting that encoding to subsequent contexts by which we study the extent that matrix is reproduced, i.e. decoded, with respect to the relationships between positions that are mapped in that subsequent context (Peterson II 2019). The value of these positions have been decoded with respect to the positions they obtain relative to that matrix, here a framed worldview given the infrastructure organizing that state, and in terms relevant to them in that context. This, recall, determines the significance that matrix obtains in that state by virtue of that emergent limit, the resources available to the state to which that matrix was transmitted.

Our use of eigen-vectors/values below is closely related to the concept of a power score. Much like analyses of 'power relations' (Foucault 1975) eigenvectors model the same relation between components despite transformations along a basis axis making the system appear different at scale. An eigenvalue is the factor by which these relations are replicated across intervals along that fixed axis. A scalar value subtracted along the identity diagonal of a matrix can be factored out from a characteristic formulation of the determinant of that matrix, following Leibniz' rule for the determinant of a matrix and the fundamental theorem of algebra. Recall making sense of this in cultural studies through encoding/decoding to norms to institutions to characteristic infrastructure as an indexed relation between those former concepts. There is nonzero solution to that matrix, here the ± Boolean value that determines if that compressed matrix, once decoded, reduplicates the relations that it encodes, if that value combined with the difference between that matrix and the scalar value of the identity is zero. The cross product of that matrix with the scalar subtracted from the identity diagonal results in a formula where that scalar (=power score) can be abstracted algebraically.

The projection of these compressed matrices represents a norm that implies a power-relation indexed to a knowledge set where differences between the first and second state represent a qualitative measure of power because it references, through exponent levels of unobserved priors, what licenses it. The exponential references the initial state over however many times its projected over time. For us, matrices are categories related in a particular way such that certain objects obtain those categories and others do not. Those that do obtain, obtain some significance in that state. A measure of hierarchical determinations is had by one system's entanglement with another. Information from one provides some information regarding the other. As such, there's no rote way to explain an occurrence's significance upon immediate observation but once an explanation is developed, we can check to see if that explanation obtains given the model we abstracted from that experience (Aaronson 2011).

The reproduction of an $n \times n$ matrix over however many iterations is a test of its extension for and to the benefit of some prior position. "Power is both created by, and summarizes, the interactions of a society. . . manifold interactions within a social group lead to hierarchy of status that bears some – but often not very much – relationship to the original intrinsic properties of the individuals themselves." (DeDeo 2016, 7)

3. Prior Models

Matrix notation allows us to study not only the formation but the (infra)structure of an immaterial yet objective basis for organizing states of affairs that produces in material (=consequential) effects. This amounts to what modes of expressions, not just speech but behaviors, are licensed given the constitution of that state. How norms evolve and emerge under different names but remain functionally equivalent can be modeled through a discussion of the breadth and depth of that matrix which, ultimately, is a discussion of the complexity of that encoding.

With regards to 'social consensus' or norms, breadth is a measure of the power of a particular node in that matrix with regards to one's belief about that node across the contexts wherein its position's retained. Depth is a measure of the 'higher-order' references this node calls upon across contexts, i.e. beliefs that others have about what others hold about that node. The former tracks emergence, the latter tracks evolution. These concepts were highlighted in the prior model when discussing the identity paradox, the extent to which an identity must call on what it's not to substantiate what it is (Peterson II 2019; 2022). Depth measures a qualitative, exponential because referential, measure of the number of justifications required for an identity to claim its position within that matrix. Recall the dyadic binary regress noted in section 1. These non-exchangeable encodings emerge at a $n(n-1)$ rate of dyadic interactions (is-x because not-$y\ldots$) required to maintain the influence of a particular power score.

In previous models, if a matrix is composed of $n \times n$ positions, then from Peterson (2019; 2022) and P.F. Strawson (1959) each position in that matrix would name, i.e. index, a category determining the conditions in which that name can be applied. From this it was possible to show that a name's applicability can be represented as a vector across contexts of assertion. Vectors having direction (=extension) and magnitude (=significance), the movement agents are allowed across identity positions relative to others can be reformulated in the following way.

3.1 Vector Components

A name's initial application to something obtaining a node in our matrix indexes the conditions in which it applies. Reference is not necessarily to a thing but to the conditions in which that name is licensed, hence the same name can apply to multiple entities and we can account for the evolution of that entity over time despite changing features. An operator's named, i.e. identified, position is expressed by an encoded relationship between features that becomes the object of a proposition or description licensed within that state. Philosopher Delia Graff Fara has done important work on this in "Names are Predicates." At this point we take a name N and set it identical with the conditions between the features it indexes – e.g. birth, parentage, behavior, appearances, et al – as W.

That name's appropriate use across contexts indicates that some object has obtained the property of satisfying the conditions indexed by that name. Using ordinal logics, a name's employment cites prior contexts of use, contexts indexing previous conditions satisfied by that name and following its initial 'baptism' indexing its first use, back to those original conditions (Turing 1939). In this way *N* at its zero indexes the function of its application, $N'=N_1$ subsequent use, $N_1'=N_2$, etc. This line of citation i) ensures reference obtains regardless of where in the chain one's introduced in the history of use; ii) determines if an instance lies outside of that line, whether it is incorrect; or iii) if that use is licensed in these conditions, marks this instance as a creative use of *N* via functional composition – output becomes input to next use which functionally relates the contexts licensing both.

The most recent use of a name is a function of current conditions. *N* applies just in case these conditions follow from the conditions in which *N* originally applied. A function $R(N)$ maps instances within that line from a domain of selection to a sub/co-domain of projection, i.e. taking the product of this instance with the naught instance and mapping that solution into values $\{\mathbf{0,1}\}=\{0, \{0\}\}$={invalid, valid}. Membership in that line or not can be determined by representing the functional content of *N* when *W* conditions arise for the index of *N* at its zero is the function determining the domain in which it applies. Use of that function is an extension of *N*. As such, *R* iterates a sequence of that functional content composing instances of *N* and thereby constructing, i.e. providing the modes of expression, of *W.*

This formalizes the three components of a state-vector

$$\langle W, N, R(\mathrm{N})\rangle$$

organizing what we know to be the case in the current state.

3.2 Matrix Notation and States of Affairs

Traditionally, an equation is translated into matrix notation by matching each of its components, from left to right, with a corresponding Cartesian coordinate *x,y,* etc. mapping to a node in a column, first to last respectively. We've listed components in nodes horizontally, making the columns the axes, e.g. all *x*'s in a column. Within these axes, relations between multiple equations' components are mapped, treating relations as the object of study. Therefore, we map/model relations/interactions between types of components across the vectors framing a possible state of affairs. As a result, we do not assume any interaction or that we've mapped a state, in all its possibilities, prior to analysis. Vectors A,B are represented row-wise, with their respective components *w,n,r* horizontally. Thus, A,B are the axes plotting relations between their *w,n,r* components with *W,N,R* columns. So framed, the shape of the interaction producing the next state of affairs is identified/determined between A,B.

If we were to write A,B components column-wise, we would be mapping the determinate of the operation we model along the matrix diagonal. However, we plot the extension of the types of components, e.g. from *w* to *w* in the first column and *n* to *n* in the second, in a 2×2 matrix. This provides the *x*-component of the state through which the interaction is being tested. An operation *r* provides the *y*-component of our interaction and through their combination, we map an interaction through these phases or 'gates' – i. e. *n* given/in *w* – giving us the magnitude and direction of the change in the state of affairs when these two frames converge or diverge, i.e. a *z*-projection.

What we show in our 2×2 matrix is the *w*-component that's to be tested based on an assumed interaction between A,B states.[1] Of course, a world is spatial and therefore assumed to have *x,y*-components internally. However, externally those components are part of the same world or *x*-coordinate to the test. Thus, the 2×2 matrix becomes an *x*-coordinate in the test system. That component is transformed by a complex *r*-component that becomes our *y*-coordinate. That operation provides the resulting *z*-coordinate of the test as the root/route of the square of the difference between coordinates. The length of that projection

[1] The cross-product of the *w*-component renders the *x*-coordinate positive, i.e. (0×0)-(1×-1)=1. This models the projection of a discrete measure of a continuous path into a framed reality that's then multiplied with its conjugate in the complex domain containing the possibilities implied by that state's evolution. This is of note because if there are multiple ways these affairs could have evolved into the state we measure, then there are *n*! possible states being filtered through each 'gate,' here component, i.e. phase, of the formula from which our matrices were built. A path could have taken a different 'turn' at each gate for in reality a path is never straight with space itself having a shape, containing obstacles. The sum of these discrete probable measures with their alternatives projected through these frames/gates is realized as a probability distribution. The amount of change in probability at each 'turn' determines the shape or the curvature/route of the path, the square root being the average or best fit of all the turns taken from one point to another, eliminating the most improbable turns as they cancel each other. The number of phases increases the possible ways one could have arrived at this discrete state; while passing through each phase, i.e. evaluating these possibilities through frames/gates, reduces those possibilities to those probable given our framework.

As such, we have to account for that growth and its conjugate, i.e. alternatives. Coordinate-wise, the factorial in one direction may develop above the real line with its conjugate below, giving us both parts of a 'wave,' i.e. evolutionary path, we use to describe the development of a state of affairs. When these probable discrete projections are taken together, we normalize the path connecting them by taking the root/route across that distribution, setting a parameter or a frame on alternative paths so as to measure the probability that the state we seek shows up as expected given what part of that curve appears in our frame. This is equivalent to the norm multiplied by its conjugate (= the true value of the formula squared = the root/route of the square of the *x*-component in addition to the *y*-component) we detail below. Geometrically, the above gives the distance from the tip of a peak to the bottom of a trough along the probability density of every point in the path to our current state. Our transform flips the conjugate up above the real line, making a curve. The n! in one direction and its conjugate in the opposite direction, representing both possibilities of development, gives us a complete phase showing a pattern in the movement. What we see it as depends on how/when/where in that phase we frame it.

measures the magnitude of the transformation. This is why we treat each component as a 'gate' through which interactions pass rather than a matrix representing a complete mapping of the state of affairs upon which operators act.

Below, the w-component as the x-coordinate translates into the traditional notation as the x-coordinate in the first column is positive with the y-coordinate in the second negative. This means that a projection from y to x has occurred, registered in our experiment. As our matrices are Hermitian, interaction occurs only when a component registers on each node of the diagonal of the matrix. Therefore, whether we choose the orthogonal notation we did here or the traditional notion with *x,y*, etc. column-wise, we obtain a symmetry between notations which is what one wants in a physical interpretation of a state of affairs.

3.3 Dyadic Interactions

To formalize the 'dyadic interactions' noted above, i.e. the 'encounter' as formalized by Peterson (2019; 2022), we must represent the interaction between these states given the infrastructure in place in matrix form. Previously, formalization meant we 'stack' these vectors. Given two vectors A and B, our matrix is represented

$$A\colon \langle W, N, R(N)\rangle$$

$$B\colon \langle W, N, R(N)\rangle$$

$\langle W, N, R(N)\rangle$ is equivalent to $\langle\langle W, N\rangle\ R(N)\rangle$ according to work done by Gödel on encoding sequences. Choosing between $\langle\langle W, N\rangle\ R(N)\rangle$ or $\langle W\,\langle N, R(N)\rangle\rangle$ symbolizes a change in basis or reference frame because, e.g., $W_A = Ncosin - R(N)\mathrm{sin}$ and $W_B = Nsin + R(N)\mathrm{cosin}$. Frame change is represented by a rotation about some axis resulting in capturing the comparative movement of the components of a vector representing changes in the relative positions available across states of affairs. $\langle\langle W, N\rangle\ R(N)\rangle$ is important here, for the first two positions set the coordinates defining axes that frame the last position describing the extension, the direction and magnitude, of a subject's points of interaction, their movement through that frame. The W represents a position on the semantic or real axis while the N represents a point on a rhetorical axis or the point of application. Formalized by cultural theorist Henry Louis Gates in "A theory of the Tradition" in *The Signifying Monkey*, the above that translates nicely into the cultural studies model based on Gate's (1988, 48) work. This provides a basis for our articulatory model.[2]

[2] Convergence/divergence of reference frames about their respect bases – i.e. axes through their origin and along which they rotate – can be accounted for using left/right-hand rules. A z-projection from y to x capturing a subject's position with respect to its frame is, in the other frame, internal to that subject, representing their frame's projection or direction/movement, in their own conditions, from x to y. The slope of z switches direction, from left-right to right-left. The cross product describes where these two projections intersect. To capture some thing from one frame where the z-axis tilts away means that we use the opposite 'hand' to view it so that now z's movement is measured toward the origin of the viewer's *x,y*-frame.

The cross product of these vectors' components formalizes the determinant of the state that emerges, C, the result of a subject's interactions within the environment framed in the above way. Movement through the environment is characterized as a vector plotting that subject's points of action in those conditions, determining the next state as the output of those actions in that environment, thereby, changing (=rearticulating) how we 'see' or the framing of that environment.

3.4 Determinants

The determinant between two or more vectors is found by an operation over the matrix they compose. For a matrix M we note each node's position as

$$\begin{matrix} & w & n & r \\ \mathrm{M} = & a1 & a2 & a3 \\ & b1 & b2 & b3 \end{matrix}$$

Their cross product composes the *w,n,r* components of the determinant vector, the result of our analysis. The cofactor expansion of the determinant becomes

$$\mathrm{M}_{\mathrm{wrj}} = a \times b = w \begin{vmatrix} a2 & a3 \\ b2 & b3 \end{vmatrix} - n \begin{vmatrix} a1 & a3 \\ b1 & b3 \end{vmatrix} + r \begin{vmatrix} a1 & a2 \\ b1 & b2 \end{vmatrix}$$

resolving to the cross product of each component as

$$= w(a2b3 - a3b2) - n(a1b3 - a3b1) + r(a1b2 - a2b1)$$

The result is a characterization for each component of the resulting state description, providing the power score of this articulation. These power scores provide a measure of the extent to which each position is 'overdetermined' in the

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Covarying with basis along x, y, and z coordinates, a single vector maps component organized along semantic, rhetorical, and projection axes determining a position within a system:

$x\,y \mid z = w\,n \mid r$

Contravarying with basis, multiple vectors form a matrix mapping the components of a system of positions. Therefore, each type of component is determined along a *w/n/r*-axis, determining a relationship between positions characterizing the system itself, providing a power score for those positions when used to organize reality:

$$\begin{matrix} w & n & r \\ x & x & x \\ y & y & y \\ z & z & z \end{matrix}$$

The 2×2 matrix models the interaction of two or more z-components where $z = x + iy$, here $n = w + r(n)$, characterizing the axis about which that position rotates, about which the *x,y*-frame turns, and converges away or towards others, defining a power relation between positions within the system described.

newly articulated state. Overdetermined being that regardless of the initial input, the same output results.

We also discover some unique relationships. For example, N=W+R, i.e. an identified position is valuable insofar as the conditions in addition to the relation that named identity obtains in those conditions. If say the value of that W component is less than that of W in one of the vectors that went into that encounter, then we can say that that component is overdetermined in the following state. Given these conditions, the difference of the relationship between components, determining the w component of the new state, goes against its prior position and, because we have formalized this interaction, we discover where this determination is coming from, namely, $a2b3 - a3b2$.

In previous models, values need not be assigned to these components for we were most interested in the relationship those components obtained in the newly constituted state. However, this required, which was seen to be a benefit, looking back at the historical record to encode each position and, most importantly, an interpretation by the analyst regarding the relative position of each component with respect to that record. We found that a top-down determination had to be made prior to our test and justified when modeling the relationship between the vectors going into the encounter. Our verification of the output against the historical record or our current experience of whether the expected outcome obtained may allow confirmation bias.

4. The Concept of Overdetermination

Before moving to improve the previous model, we attend to one of the goals of ART-theory, a formalization of the concept of overdetermination. Recall, overdetermination means that regardless of output the value of a particular node within the matrix organizing our affairs always comes out the same. If this occurs, we can refer to the determinant of our product and perform an operation to see which node this is coming from. An alternative to circumvent overdetermination is revealed as an encoded possibility as yet actualized in current affairs. A contemporary example would be the increased representation of certain groups yet the structural position of that group largely remains unchanged – see Bright L. et al. on "The Stability of Racial Capitalism."

4.1 Our Current Model of Overdetermination

A few issues were left unresolved from the prior model. How do we limit the top-down assumption required to conduct our analysis? How do we surmise whether a formalization of these affairs and the alternative ascertained through the cross product would be significant if ran through that matrix once more? If we run that alternative through the same matrix, we are modeling an alternative being actualized from what is apparent. Although it was an encoded possibility prior to implementation, in the moment, the same matrix is being employed in this context organizing its affairs in a similar way to the past.

4.2 Encoding

We'll now treat these vectors as physical entities. Our inspiration comes from the sciences where we see the effect of formalizing matrices in the imaginary plane by abstracting features from the real and projecting that encoding back onto the real given certain conditions, testing to see if our expectations arise.

Encoding proceeds as follows. We find a set of conditions B in which a particular set of features s are encoded onto a matrix r_{ij} in addition to an emphasis on nodes s_i given by T. Formally,

$$= \frac{\exp\,(B\sum_{ij} r_{ij}s_i s_j + T\sum_i t_i s_i)}{Z}$$

wherein r gives us a matrix – the one created through our mechanism above – and T emphasizes specific nodes on that matrix, mapping the points of overdetermination required to reproduce that matrix given available resources. This is held relative to a normalizing factor, following the root of, 'route over,' the number of possible states rule. This rule allows us to test the distance between our projected encoding in this context and the index of that encoding in the original context. Conditional upon these points obtaining in a context successor, T emphasizes what's been projected into this context, organizing what we deem significant.

This formula correlates with phase changes in physical systems and has been applied in other subject areas. It shows the extent to which a substructure of a distribution holds insofar as it identifies the state that system is in given the conditions harboring alternative configurations yet from which this structure emerged. This correlates with the cultural infrastructure formulation in section 1. When B is high, the conditions exhibit a low tolerance for deviation from T. Minimizing B increases the influence different interactions have on each other. These measures may correspond to the 'mood' of a time indicated by, e.g., increased policing, or the decreased strictness in norms evidenced by more tolerant legislation, etc. Shifting conditions either increase or prohibit the ability for agents to interact given the matrix employed. In physical systems, frequency of interactions may correlate to temperature. Here we can say the temperature of the time or state is its mood.

We're interested in r and T's relationship. It's been shown that a minimal percent of nodes must be transmitted to produce the emphasis required to reproduce the internal organization of the encoded state in a subsequent context (Miton et al. 2022). Those nodes upon which that matrix was constructed, but not emphasized, ensure that structure is retained by context appropriate features organized around that structure. In fact, only ~10% of the internal structure must be transmitted to say that the encoding has been communicated (Miton et al.

2022). This tends to some information loss or fuzziness around organizational structure, but also the possibility to articulate new but context relevant formations from old structures.

We can evaluate norms and their evolution in this way. Constraints are pairwise according to the above. A single node's position isn't transmitted, only its relation in some structure. There are preferences placed on certain nodes in relation to others – recall the force of interpretation in our older model – to maintain the integrity of the matrix's structure in hopes of reduplicating the relationships it encoded in the future. Alignment or anti-alignment allows us to utilize a measure other than a top-down assumption in accordance with T encoded in r in the determination of the matrix. Herein the motivation for our improved model.

4.3 Modeling Emergence

There is a way to model the evolution of the relationships between positions in a matrix so that we don't need to externally determine hierarchy but study how it emerges. Above we noted that there are certain preferences placed on node combinations, describing possible routes/roots with respect to how one navigates the matrix organizing their affairs. Above, one preferred the move $a1b3$ to $a3b1$. However, the availability of these moves is indexed to particular categories which are hard to change. A pertinent example of one that's attended to below, is Race. We can look at the process of racialization with respect to markers that don't have intrinsic value but, hoping to imitate successful moves closer to powerful positions in the matrix acquire flesh, so to speak.

Given two outcomes s_i and s_j indexed to $a1b3$ to $a3b1$ respectively, and labels for positions with different probabilities associated with their exchangeability, the imitation of or the emergent value associated with a particular move toward some position within some matrix becomes

$$= \frac{s_i - s_j}{s_{max}}$$

where s_{max} is the maximum possible difference/distance between positional values in that matrix. Dependent upon whether s is positive or negative, choosing that move is now associated with a particular position. Recall section 1, choice given options provided by infrastructure is identified from another position as representative of the formers framed worldview. That position's label begins to acquire more value as more individuals, regardless of label, imitate that move despite their inability to exchange their label for the benefit of another. This innovation by Liam Bright et al. (2022), means that we can simulate how certain vectors acquire a particular magnitude as opposed to others on some hierarchical scale. This relies substantially less on external top-down stipulation. In fact, the determinant from the previous model correlates with the capacity to exchange labels within the matrix. Certain labels are shown to be overdetermined given the conditions that emerge as positions interact. Proof follows from change of basis formulations in matrix notation.

4.4 Vectors Revisited

In cultural studies, an analytic framework was developed whereby we map the structure of society between semantic and rhetorical axes (Gates 1988, 48). Coordinates within this space allows us to map relations between matrix positions, drawing out the shape or 'form of life' of a particular state. This conception is amenable to our purposes for the grid over our reality's state of affairs models what organizes experience and is how we know what is significant in the space plotted between those axes. The negative space provides a picture of the institutions allowing or disallowing certain positions implied by the structure of the larger state of affairs, giving us the shape of the object of study in relief, the infrastructure.

Physically, representing a vector's components as a complex number, with real and imaginary parts, allows us to talk about a three-dimensional frame within which that vector moves utilizing only two axes. The frame or system moves up and down an imaginary axis. A vector z in three dimensions from a determined 'origin' is represented $z = x + iy$ with $i = \sqrt{-1}$ and x = direction, iy = magnitude. As our articulatory model is geometric, this complex translates to $z = \cos(p_1 - p_1) + i\sin(p_1 - p_2)$ where $(p_1 - p_2)$ is the distance between observations in the real given the same matrix's projection across contexts, i.e. the angle of projecting a frame to organize both contexts. The 'imagined' part, iy, is the magnitude of abstraction from the real to apply in future contexts and z is the direction of the framed vector moving with respect to *x.*

With this in mind, a position along the rhetorical axis is iy and on the semantic is *x.* If in a coordinate system the horizontal axis represents the real and the vertical the imaginary, when rotating that *x,y* plane along *x* so that both *x,y* are horizontal in two directions, *iy* becomes the magnitude up the vertical that determines the extent to which a projection along the real in the x direction obtains from one phase of analysis to another*,* i.e. the area covered by the vector z drawn from some 'origin' up towards the point marked by *iy* representing the form of life of that vector across an $n!$ space. Using a Cartesian coordinate system, this can be shown as equivalent to the combination of that vector's components and the *cos*ine of the angle of the direction of travel across the real domain, i.e. the difference between projections. What's most interesting is that this interpretation geometrically resembles the same as previous cross product formulizations. The determinant of an encounter between frames organizing experience produce a vector with a real value that can be calculated as moving in a particular direction.

5. Cultural Studies Translation

Applying this to cultural studies, whether a vector is represented vertically or horizontally depends on changes between its components with respect to a change in basis, i.e. changing reference frames. If we translate ours into Henry Louis Gates' system in *The Signifying Monkey*, states of affairs would be organized along semantic and rhetorical axes. Coordinates within that space represent relationships between components – i.e. features of experience filtered through the matrix of some frame, here, the grid between our axes that's applied to organize that domain. Framed positions index locations for concepts we use to understand formations outlined by relationships between them. Coordinates represent the intersecting forms of life of vectors whose interpretive bases are relative to these axes. Framed between a rhetorical y-axis and semantic x-axis, vectors are articulated in a z-direction. For us, W references the origin/intersection of axes framing movement; R sets the angle and magnitude of the vector determining the relationship between semantic/rhetorical coordinates; and N names the coordinate to which that vector is projected from W and determined by R in the z-direction.

The 'angle' referenced above is the difference between the norm and its conjugate, which in our terminology means the semantics given the projection of some frame to organize a particular, real, set of conditions across contexts. That conjugate space is the infrastructure through which that norm operates. The $i\sin(p_1 - p_2)$ measure above gives us the magnitude of utilizing, i.e. the extension of, that frame. The distance, the displacement in physical terms, between one projection and another – the hypotenuse in our geometric representation – moves us from one index on the real line, here the semantic axis, to another. This in turn measures the significance, the gravity, in organizing real affairs relative to the frame projected.

Rotations about an axis represent reference frame changes with respect to one's orientation in that domain. A reference frame is a worldview, one's orientation to/in a state of affairs. If selection along a rhetorical line (=use) varies inversely to semantic base, the frame capturing z rotates about the y-axis, mapping changes in use value relative to changing semantic bases. A different frame means the same term obtains different semantic values. If use co-varies with semantic basis, the frame rotates about the x-axis, mapping different uses with the same semantic base. Different frames mean different terms obtain the same value or are invariant with regards to x. In both cases, semantic value is a function of use with respect to changes in reference frames. Through Gates Jr., our model considers concepts as relations between features abstracted from a domain framed along these axes that enter our reality by organizing future experience with respect to these concepts. When concepts are projected into contexts to see which objects obtain that relationship between features, we model how concepts enter our reality. Dependent upon whether your frame rotates in a co- or contra-variant manner, an orientation towards concepts connects individuals in such a way that their world-view aligns, articulating particular forms of life organized around these emergent relations about these concepts.

This brings us to the conjugate mentioned above. The conjugate of the vector components changes the sign of the imaginary component, projecting what was abstracted back onto the real and testing the extension of that vector in that plane. This proves useful for taking the product of a vector and its conjugate provides a test of our frame through the domain in the reality revealed by its conjugate. We understand this through the Pythagorean theorem. A vector tested through, i.e. combined with or multiplied, by its conjugate is equivalent to the sum square of its components, allowing one to triangulate a position in reality from various levels of abstraction, here levels of articulation. The square of the absolute value of a vector becomes the probability of that identified position given our test, i.e. its significance given that those conditions in the conjugate arise.

A few more considerations must be made before moving on. Operations on vectors through fields is how we conceive of the ability to test our theories through this articulatory mechanism. If we multiply a vector with a particular value, each component of that vector is multiplied by that same value. If we multiply a vector by another vector, then the result will be the sum of each component multiplied with its respective component from the other vector. Thus, vectors a and b such that a is the conjugate of b means that

$$\langle a \mid b\rangle \ = (a1, a2)\begin{pmatrix} b1 \\ b2 \end{pmatrix} = a1b1 + a2b2$$

This is key, for $\langle w,n,r\rangle = \langle\langle w,n\rangle r\rangle$. Given two vectors, we can treat a test of a relationship between identified positions, i.e. ascertain the state of that system, in a field of observable measurements in the following way

$$W|R\rangle \ = \begin{pmatrix} w1 & n1 \\ w2 & n2 \end{pmatrix}\begin{pmatrix} r1 \\ r2 \end{pmatrix} = \begin{pmatrix} w1r1 & n1r1 \\ w2r2 & n2r2 \end{pmatrix}$$

where W represents a matrix such that it is the conjugate of the relationships mapped by R when transposed. Call this M_H – see below. The diagonal $w1$ and $n2$ remains the same but $n1$ and $w2$ switch signs yet that operation doesn't change the values of those positions in the matrix. The diagonal is the identity of that matrix, which brings us back to our eigen-vector/value formulation. The result is a vector that is the same as R but with a value assigned, usually ± 1, telling us whether that relation obtains in a field organized by W.

R is now considered an eigenvector. The testability of frames is able to be considered as part and parcel of our notation. Statements within ART-theory, theory in general, are only significant insofar as those frames are shared so that when tested in the real, if a functional equivalence between objects comes about, then we can say that we understand which concepts organizing our experience are at work. These concepts are not things in reality but are nonetheless 'real' in the sense that they structure what can be said is real, what's apparent to observers with recourse to the same tools, across frames and amongst individuals. This, despite those things' with these properties' superficial appearance.

So when looking at an interaction between two frames ⟨*W,N,R*⟩, we understand that ⟨*W*| represents the frame projected onto experience, |*R*⟩ the relationship between positions really obtainable through *W,* and |*N*| the real value associated with that position, given *W* as a function of *R.* The Eigenvector, i.e. a vector projected through a field and producing a measure of that vector's applicability in that domain, provides a measure of social power – see DeDeo (2016). So the determinant of the cross product of the interaction between two or more vectors is a measure of the significance of the relationships obtainable in the newly constituted conditions that interaction produced. Identification by name is an immaterial yet objective projection onto reality whose use across contexts is modeled by the vector z formalized above. Thus, z corresponds to *N*=W+R, organizing experience under the concepts employed. The value of *N* is a function of its use given certain conditions*.* Conjugation reflects the assumed identity of the state being tested back onto the real line, thus we say that *W* is acting on a relation, here vector, *R* through *N*.

5.1 Matrix Identity and Determinants Revisited

ART-theory measures indices along the rhetorical axis that license what is or isn't expressed in the semantic reality because that determinant frames the probability of what is or is not available to utilize rhetorically, i.e. to express one's position. This organizes the reality we have access to via the mode of expressing one's position and identifiable by others along that axis utilizing that frame. A matrix that is identical to its transpose represents a field through which a relationship between real positions can be tested. As a result, when we formalized the determinant of an interaction between different frames' components, we discovered a way of testing which relationships in that determinant are possible given the structure of the field. We test our determinant results against the space that has been mapped between our semantic and rhetorical axes, obtaining a value indexed to the positions mapped by each ⟨*W,N,R*⟩ frame and calculating the probability, $\pm$ on *R*, associated with that determinant.

We test determinants by treating, ⟨*W,N,R*⟩=⟨*W N* | *R N*⟩, for *R* is a function of *N.* Provided with the determinant of an articulation, our experiment will give us the probability that that determinant obtains given inputs from the archival record understanding that ⟨ *W N* | is the conjugate of |*R N*⟩. So,

$$\langle W\ N \mid R\ N\rangle\ = (W^*\ N^*)\begin{pmatrix} R \\ N \end{pmatrix} = W^*R + N^*N$$

and

$$|W^*R + N^*N|^2$$

is the probability of those relationships obtaining given the inputs.

As encodings of the positions in the matrix are derived from the archive, our set up allows us to test derivations in our current state against what we thought the historical progression we modeled told us, therefore testing our theory as an explanation of current affairs. If results are positive, the structure of our matrix does map the affairs; if negative, the relations might be the reverse or inaccurate. Remember an $n \times n$ matrix encodes a state of the field. Through the cross product reduces that state to the determinant we formalized above as a hopeful explanation of the power dynamics at play in that field (DeDeo 2016). That $n \times n$ matrix represents n to a power, n insofar as the complexity of the system described could be reproduced. That power score defines the number of times that structure cites its means of production with each application. Reference being quantitative, the extent to which that organizational mechanism obtains is a complex quantitative measure in the moment, for in situ one does not have access to the weight that encoding carried in prior conditions, only in their context after application. Following this reasoning, we map the growth or spread of similar relations across domains. The reduction of this function to the determinant can be used to abstract these enabling conditions. However, reproduction in the next context is only an approximation of the first.

5.2 A Note on Archive Translation

A quick note on translation is necessary. Some detractors of theories such as these will claim that a method like Saidiya Hartman's critical fabulation (Hartman 2008), for example, licenses too much. However, it has been shown that much scientific inquiry operates in the way outlined above. One abstracts a relation between features of experience, indexing that relation under some concept, and projects that index, now encoded under some name, into different contexts to test the extent to which that concept maps relationships between the features of things in conditions seeming to follow that initial context. It is through this process that we can say that that concept obtains in those conditions. The thing that obtains that concept becomes an object (of thought) whose function in our framework is now relevant to and whose operations within those conditions is explained by that concept. Without a guiding theory, nothing will come of our endeavors. Hence the inextricable connection between theory and practice. Just as well, given our experiments, our theory updates over time. The question of whether a theory is of the world does not arise for it's abstracted from experience and updated as we test those abstraction's extension to objects of subsequent experience.

Ramsey sentences provide a model of this methodology. Taking archival observables, i.e. sentences in the archive, and summing those relevant to the concept we are developing, we then remove what non-observables we utilized to make sense of that data, and replace those non-observables with variables, leaving us with a framework we utilize in experiments to test the extent to which that framework obtains across archives of the same period. Here, that frame is in matrix notation. For our purposes, the 'Ramsey' sentence is our matrix, the values we use to index the nodes of that matrix, for Ramsey the open variables of the sentence, are the concepts we wish to test via this articulatory mechanism. We take the concepts we wish to test and see where and when this frame applies across archives documenting similar conditions. This becomes our *R* function, relating current observables as instances of those concepts by way of the objects that matrix captures when applied, i.e. across *W*'s with each *N* application. This tests the extent our concepts apply and maps their evolution over time and up to now.

A Kolmogorov complexity measure of that theory can help us bridge the gap between qualitative and quantitative measures. Taking our initial sum of descriptors, the extent to which that sum can be reduced without losing the relationship these concepts encode gives us a qualitative measure of the complexity between frameworks with respect to the quantitative measure of steps taken to obtain a solution that translates the output from one frame to terms relevant in another. The more complex a theory, the less likely we can reduce this initial framing. Testing the efficacy of that theory in explaining phenomena, i.e. its use value across contexts, we obtain a qualitative measure with respect to the quantity of steps it takes to translate that theory into a feasible option that aids to describe day to day experience or the record. The less reliant on interpretative devices the more direct the method explaining phenomena; the more steps to interpretation, each step citing the initial frame, leads to an exponential process that may not provide a direct solution. However, if a solution is provided, it can be verified with respect to a more direct frame.

6. Testing a State Model

We cannot always obtain values from the archive directly, e.g. certain increases or decreases in economic measures. Nor can we directly ascertain probabilities from historical occurrence, for what obtained there has done so with certainty with respect to that record. It follows that we cannot plot probabilities directly regarding our reading of our actual state, for we occupy that state. However, we can plot payoffs, i.e. the results of bargaining for position within particular conditions organized along some matrix (Blumer 1958), and the associated costs of a mode of expression's output and its value in those conditions (Bright et al. 2022).

With Bright et al. (2022), it is now possible to implement our method without having to stipulate a top-down relationship when stacking our two vectors to compose a matrix that organizes our reading of archival records. We can utilize a bargaining assumption tied to obtaining certain positions relative to this matrix. The imitation of a particular identity *N* reveals how relations of subordination and dominance emerge. *N*-tags, whose function are determined by *R* given frame *W,* no longer have intrinsic value but acquire value as a function of the matrix employed, i.e. within which it operates. The probability associated with changing one's tag defines the system, not the tag itself. We don't have to invent these probabilities. They are defined by how the system has determined their value.

Historically, it is extremely difficult to become White in the U.S. but very easy to be considered Black, if and when convenient (Bright et al. 2022). Tags on payoffs provide N and the difference between N's is ascertained by how they're licensed to operate, i.e. R. Game theory allows us to plot payoffs as a function between semantic value and use within a state organized by our matrix. Given the coordinates of a particular value and use, choices available to an individual given that matrix are found in the shape drawn between each option in that plane. The value of choices in that area represents the significance of utilizing that option while the intersecting projections between options drawn by lines from their defining coordinates provides an equilibrium above or below which one judges payoffs. The value of a position in that space is calculated in the usual way, by taking the root of the sum of the square of the differences between those coordinates, thereby locating one's position in or out of the space contouring the maximum difference between options. The difference between payoffs with respect to the maximum difference gives us a measure of whether that choice will be imitated by others.

6.1 Applicability

So far, we have shown how the determinant of a matrix compresses that matrix into a form that gives value to the relationships of subordination and dominance encoded by that frame. By utilizing our updated model, we can test the probability that that matrix's relations are reproduced in the contexts, i.e. subsets of the real, to which it is projected. Tests run that vector through the relationship between the positions identified in that frame, the result being the squaring of the value obtained. This models the establishment and resilience of the norms organizing states of affairs.

Norms institute group beliefs. A 'coursed-grained representation' or 'lossy compression' for which we need not all believe the same thing, but by which that norm constitutes and guides individuals that coordinate around a particular object of experience and jointly act – see DeDeo (2016). Norms are reinforcing, allowing us to parse experience by determining what acts are available (=relevant) given current conditions despite alternatives licensing acts falling outside their purview. In highlighting certain features of experience and guiding action, material changes to overall affairs inevitably occur because individuals are compelled to interact with the conditions themselves in virtue of what comes under the purview of the matrix organizing what they deem significant or not in those affairs.

We can now show how to apply our model while minimizing assumptions. Particularly, the top-down, dominant and subordinate, assumption made *before* in Peterson's (2019; 2022) analysis with our only recourse being the archival record. By checking the internal structure of our framework as well, seeing if it's viable with respect to the record, we find out whether our theory falls in or out of the space noted by the matrix organizing our mode of inquiry in the first place.

Values are assigned to *W,N,R* with the scale 1/2, 1/3, 2/3. These encode equality, subordination, and dominance respectively. This does away with one of the issues from the prior model. The notion of levels of articulation is replaced by movement of position within the matrix organizing that state. Rearticulation, then, is just a phase shift, sometimes leading to cascades of phase changes in the emergent hierarchies of that state of affairs. We retain the ability to map how these relations evolve in the model itself. Top-down determinations are not as detrimental to our findings and we can discuss the inheritability of position as well, a concept key to the study of Race, racialization, gender, and class.

Given the formalization of imitation, if an outcome is positive between options, that position is encoded as 1 in our test; if negative or zero, it is encoded by 0. This scale tells us whether the theory obtains given the conditions mapped and tested. What is most interesting about the model proposed is that across iterations, we can model the evolution of positions over time before testing the theory in a singular instance, leading to unexpected effects based on our initial reading. Herein lies the value of an articulatory model. A test of a singular instance may just turn up results we already know given the record, running the model over time may provide explanations on how we got here.

Finally, recall the 'power score' coefficient to each component above. Each coefficient associated with a factor of the expansion gives you a value associated with the probability that this component's inner structure will be reproduced in the context under investigation.

7. Case Study

We'll explore a case study to examine how this method is applied and to discuss the differences between the old and updated model. Our methodology provides a causal mechanism for a functional explanation of the articulation of relations of subordination and dominance in accordance with Stuart Hall's program. First, we'll discuss a control study that matches the historical record to test the system itself, then we'll conduct an experiment testing an extension of its application to discuss cultural appropriation.

7.1 Previous Model's Application

Consider the example in Peterson's *Black Thought* (2022). Two mappings *a,b* such that W=U.S., N_a=American (A), N_b=African-American (AA), R_a=White, R_b=Black. Black and White can be used as signifiers of identity, but here are nodes in the matrix organizing what positions are attainable by individuals (Blumer 1958). Black-ness, then, can be expressed under different names as a function of that name's (=identity's) relative position to others in some state description. Our matrix becomes

$$M_{wrj} = a \times b = w\begin{vmatrix} A & White \\ AA & Black \end{vmatrix} - n\begin{vmatrix} U.S. & White \\ U.S. & Black \end{vmatrix} + r\begin{vmatrix} U.S. & A \\ U.S. & AA \end{vmatrix}$$

Which position assumes the top and bottom row is checked against the historical record. If we're testing the extent to which White supremacy obtains in the U.S. context where 'other persons,' according to the Declaration of Independence and the U.S. Constitution, are appropriated only 3/5 status or not recorded at all, we find the emergence of this structure uncontroversial.

Resolving our matrix we find the same relations of subordination and dominance that we find in the historical record in the cofactor expansion of that matrix's determinant. M_{wrj} solution is calculated by

$$= w(A(Black) - White(AA)) - n\big(U.S.(Black) - White(U.S.)\big) + r(U.S.(AA) - A(U.S.))$$

How White supremacy overdetermines the position that Black's obtain as a function of American identity is clear as White overdetermines the output of African-American identity in the world structured by this matrix. This produces a context wherein Black's feel it necessary to produce an identity translatable in White dominant structures through a hyphen-operator, imitating what other nationalist identities have done in a U.S. context and becoming African-American (Wilkerson 1989; Page 1988). However, this time it's a continent translated into a singular identity. The issues of universalist identity we find in our *r* component. The state overdetermines that identity. The U.S. determination of the output of Blacks in the *n* component produces conditions in which the output of U.S. affairs is a function of White stipulations – see Quijano (2000); Von Eschen (2004); Mignolo (2007). "Categorical systems are crucial to grounding inequities." (Bright et al. 2022) What's most interesting is that an identity's value is a function of the relation an individual obtains, their position in the state structured by the matrix above, not in the name itself.

7.2 Updated Model's Application

We can compute the value of each component and, therefore, the overall plausibility of the determinant utilizing some of the methods explored above but unclear in the previous model. If positive, the conditions indexed by the determinant, representing the compressed matrix projected onto the real line and organizing those affairs, are reproduced in the following context. If top=2/3, bottom=1/3, and equity=1/2, then W-components would be 1/2 because in the same state; White=2/3; Black=1/3; and depending upon whether the relation *N* obtains in W is at parity or not, we locate the overdetermination coming from *W* in the first case or *R* in the second.

Issues regarding top-down assumptions and confirmation bias remained, for we already knew the outcome although the theory of that outcome was confirmed through the experiment above. Provided our articulatory mechanism, if the relationships mapped by our determinant were incorrect, then the outcome of the determinant would have failed, contradicting what the archive showed.

What, then, would a positive result mean regarding the organizational structure of that state? Either the matrix obtains, or it does not. This uncertainty communicates some information but is not as helpful as we'd like.

We found that Black occupies a different world position after that encounter. But to what extent is it overdetermined given the model? Let's return to the binary encoding that was used in Peterson (2022) with respect to the previous model but only regarding the level of development or hierarchical position of an instance of articulation. Let us also dispense with the top-down assumption in as least a controversial manner as possible. This was only slightly ameliorated by assuming values 1/2, 1/3, 2/3.

Assuming our updated articulatory mechanism's interpretation

$$W|R\rangle \;=\; \begin{pmatrix} w1 & n1 \\ w2 & n2 \end{pmatrix}\begin{pmatrix} r1 \\ r2 \end{pmatrix} = \begin{pmatrix} w1r1 & n1r1 \\ w2r2 & n2r2 \end{pmatrix}$$

with values W=U.S., N=American (A), N=African-American (AA), R=White, R=Black, we encode this new matrix using 1=dominant, -1=subordinate. We assume 0 for parity, as no effect still contributes to the maintenance of the same structure. What we're interested in at this point is that structure's reproduction. If two positions are both +1 or -1, we can assume parity. We retain 0 for testing.

What we test is represented by a vector spanning the difference between the positions White and Black given the organizational matrix in which they operate projected onto reality (=the real line qua semantic axis). Our use of complex notation highlights a probable position on a plane organized by this matrix given the maximum difference (=significance or in this case amplitude) between the identity indexes projected onto that plane. The x-initial state in addition to y-choice passed through the i-matrix organizing that field, giving us the probable result of the articulation. A coefficient's index along the i-axis gives us real measure upon projection.

Encoding our experiment, we assume the following

$$W|R\rangle \;=\; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} r1 \\ r2 \end{pmatrix} = \begin{pmatrix} w1r1 & n1r1 \\ w2r2 & n2r2 \end{pmatrix} = \begin{pmatrix} w1r1 + n1r1 \\ w2r2 + n2r2 \end{pmatrix}$$

making sure to note that these values are complex. This is also a compressed expression of a Pauli matrix for which the differences between the values for $n1$ and $w2$ are represented by 0. The value for $n1$ and $w2$ are set to zero because they are not being tested.

We wish to see if a Black identity is subordinate in a White world. We are testing the observation that a Black identity under the name African-American is subordinate in a U.S. state determined by White supremacy. A parity measure would mean that by testing b in a, e.g. $\langle a|b\rangle = |a|^2 + |b|^2 - 2|a||b|\cos(x)$ for the value would be 1/2, i.e. shared. However, assuming that the articulatory process is iterative, therefore we sum the respective components to obtain the next iteration, e.g. $|a\rangle + |b\rangle$, if a state involving parity passes through to another involving parity, we would get >1 probabilities in [0,1]. E.g.

$$\langle a \mid b \rangle = (1 \;\; 1)\begin{pmatrix}1\\1\end{pmatrix} = 1 + 1 = 2$$

Taking the square of the true value of that output would give us probability 4 in the interval between 0 and 1. This brings back our root *N* rule. Normalizing passage through identical states, if the true value of the positions in the columns in the matrix being tested are identical, each input is multiplied by $\frac{1}{\sqrt{n}}$, here *n*=2, so that we can maintain an iterative, i.e. additive, process of articulation. While cross products remain, the relationship constituted by the compression of the matrix organizing affairs makes

$$\frac{|a\rangle + |b\rangle}{\sqrt{2}} = 1$$

Moving to our experiment, we'll test if White maintains dominance as abstracted from the record – (1 0) – or if it is possible that Black is dominant – (0 1).

$$W|R\rangle = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

from which our assumption is reproduced. A positive value accompanies our test serving as an explanation for why our matrix assumptions are most likely the case. Testing a counterexample, i.e. White subordination,

$$W|R\rangle = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & -1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$$

from which we find a negative coefficient must be abstracted to resolve, i.e. explain, the disparity between our test and the result. The counterexample is most likely not the case.

Our assumptions decreased because we didn't have to interpret differences outside of the record, i.e. assume values for the record to determine the relationship we wish to explore. We tested *R* components to see if they obtain our matrix. If not, there's a mistake in our interpretation. If no other evaluation can be found, then this is the wrong interpretation.

Once our interpretation is justified, we can then test the determinant provided by another study, i.e. as the assumed output of the use of a frame in the archive or an apparent (=assumed) historical result. Using,

$$\langle W, N, R\rangle = \langle W\, N \mid R\, N\rangle$$

we test our new result against what we would have obtained using our old model, where a matrix *W* determined by *N* can be shown to exhibit *R* relations between its components. This can be reduced to the new system by encoding the positions in our matrix with complex representations of the relative values 1/2, 1/3, 2/3 we used before. In computing the determinant we now calculate the viability of the matrix abstracted for testing also. The square of their absolute value represents the probability that this configuration obtains once more without having to assume a top-down representation, only ensuring that the matrix being tested is equivalent to its transposed conjugate.

When testing, recall that the angle of projection in our geometric model translates in this system as testing the displacement between projections, i.e. the significance of organizing the real relative to the matrix projected between contexts. Conjugation means that the *N* has changed signs with respect to the zero of the vector ⟨*W,N,R*⟩. Testing the determinant in the way outlined above allows us to tell the difference between the assumed organizing structure within the determinant and the outcome of the articulation computed in our model. In other words, the extent to which the determinant reflects the real described by the ART model.

Our test follows from how vectors are represented in complex states, for x=$zcos$(difference) and y=z($isin$(difference)). The projection of our matrix determines how the real is organized becomes the rhetorical with respect to the semantic like in Gates (1988, 48). Solving for z is equivalent to solving for the root of the sum of the difference between coordinates. That difference, i.e. angle of projection, is solved by the difference between the real index relative to the semantic projection for each coordinate. This is equivalent to the inverse of the tangent of the relation above. We find that many of these polar coordinates obtain the same value when projected, adding further evidence in favor of our interpretation as it can handle the difference between how across context meanings may vary while representations remain the same and the inverse.

7.3 Alternative Reading of Appropriation

Consider the case of appropriation. The dominant imitates the subordinate. Appropriation is measured by a movement along the i-axis by which y-significance measures the extent of imitating others, testing how far the z-value projection retains that value along intervals across the x-real axis. Here, we test the extent that a rotation around the i-axis, i.e. the convergence of positions relative to the projection of a frame, obtains the same value. Convergence of frames represents imitation. Rotation around an axis represents a change in frame whereby appropriation becomes the extent to which movements, i.e. articulations, are mirrored. It seems that if White imitates Black, then there should be a shift in the power score. White's expression, although dominate, would begin to resemble Black's; their positions would converge across frames. Testing this assumption, if ⟨0, 1⟩ = (Black power shift), then,

$$W|R\rangle = \begin{pmatrix} 1 & -i \\ i & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -i \\ -1 \end{pmatrix}$$

However, recall, i=0 on the x-axis – see Pauli notation. This gives us a negative value abstracted from the test. Testing ⟨1, 0⟩ we get a positive value. Despite appropriation, White giving ground does not lead to an exchange in power score. How?

Our reading follows the *cosine*-generalization rule. Appropriation can be formalized as a rotation of frame towards a point in the domain. We must see whether upon rotation, the same principles governing the triangulation of position within a state of affairs converge towards equivalence. Geometrically, this proves to be the case, albeit not at scale – discussed below.

Say that the angle of rotation is the difference between projections of a frame onto the real, i.e. a comparison between two contexts organized by the same frame such that we obtain a y-rhetorical measure given x-semantic position. The articulation of positions within a frame is equivalent to a phase change, i.e. measure of the difference in output within each frame across contexts. The measure of articulation given appropriation seems inverse to the change in power score when the expressed movement of a position relative to others is passed through the matrix organizing the affairs of each context.

It follows that if we know this difference and we measure a distance between positions in reality from what we may call an "origin" and some coordinate $(a, 0)$, we can say that in that plane the coordinates of the point from which the projection is made is $(bcos, bsin)$. Following the Pythagorean theorem, we can show that the root/'route' of the vector articulated through that frame is equal to the root of the sum of the difference between a and $bcos$ squared in addition to the square of $-bsin$. This resolves to $c^2=a^2+b^2 - 2abcos(x)$ where x is the difference between projections, i.e. the angle of projection of a frame across two instances in the real. This generalization allows us to see that a measure of convergence or divergence is implicit in any triangulation of position. Our test of appropriation measures the convergence (=imitation) between positions across contexts.

In sum, formalization of the $2|a||b|cos(p_1\text{-}p_2)$ rule draws p_1 and p_2 as lines of projection of reference frames across contexts on the real. Given an observation of appropriation, then, we measure the significance of that imitation in relation to the frame organizing the reality within these contexts.

From context to context, we may witness a phase change, a jump from one space in the domain to another, such that it appears that a Black identity is gaining traction against its overdetermination. However, despite the N score for Black's increase, the R score decreases. Black is imitated but its position cannot be exchanged for $N=W+R$. Upon change of frame, the capture of the benefits of certain positions models appropriation but the principles governing position remain equivalent making the outcomes appear different, superficially, but leaving structural overdetermination intact. This could explain why Blacks are seen as being represented culturally and politically despite the matrix determining the value of the output of their affairs only shifting slightly (Mineo et al. 2018). We also find a unique interpretation of the concept of interest convergence (Bell 1980) in which the dominant will cede position, even in the act of appropriation, (Taiwo 2022) but only insofar as it benefits the frame in which the dominant's position is maintained. In effect, the above models the racialization of labels. A particular matrix or 'gate' may allow for a specific mode of expression or license individuals to move in particular ways, but due to its indication of a particular position, label exchange is hard to come by. Therefore, certain labels remain under the bargaining limit despite superficial representation.

8. Summary

From our appropriation discussion we see the power of this alternative to previous articulatory mechanisms as well as a marked change in how we discuss socio-cultural and political processes of articulation. We progressed from a hierarchical explanation to an explanation of the evolution and dissolution of hierarchy over time within the domain. If the emergence of norms between n-states entails $n!$-possibilities in a framed domain, then normalization via $1/\sqrt{n}!$ provides the plausibility of our analysis of the state articulated from initial conditions to a final state. ART-theory studies the means of traversing that field whose infrastructure frames a set of possibilities. Our mechanism can be summarized by the following.

From $\langle W, N, R\rangle$ we treat $\langle W, N\rangle$ as a matrix through which $\langle R\rangle$ is tested. Every matrix is derived from the following

$$M_{ART} = \frac{1}{\sqrt{2}}\begin{pmatrix} \delta & \delta - i\delta \\ \delta + i\delta & -\delta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

and shown to be significant from the generalization made possible through our alternative reading of appropriation. Each δ-value is considered the resultant determinant of a prior articulatory phase. We must normalize this value, $\frac{1}{\sqrt{2}}$,for considering each position as distances traversed across quadrants of a plane, a sort of superposition of alternatives result, each equally existent until measured. Recalling our eigen-value/vector=power score formulation above, the cross product of this matrix with a vector we run through that space produces that same vector with a power score that can be abstracted from the result algebraically. The proof comes from the fact that if a scalar value is subtracted from the identity of that matrix, the cross product of that matrix gives us a nonzero value just in case the difference between that matrix's encoding and that identity multiplied by that value is zero. As such, a power score p can be abstracted from tests results when

$$|M_{ART} - pI| = \begin{pmatrix} \delta - p & \delta - i\delta \\ \delta + i\delta & -\delta - p \end{pmatrix} = (\delta - p)(-\delta - p) - (\delta - i\delta)(\delta + i\delta)$$

$$= p^2 - \delta^2 = (p - \delta)^2$$

where p is the value that must be abstracted to ensure that that resulting vector remains the same as the one prior to testing. It follows that however large the matrix, if it satisfies M_H, then we multiply the terms of the identity diagonal from which we can abstract a value for the vector that results. Each phase-structure derivable from this matrix represents a rotation of frame towards or away from observables across contexts. Rotations simulate imitation and, therefore, bargaining for position within these structures as a function of appropriation. Our generalized notation, in accordance with treating each position in a complex state, means that through this matrix we can represent a three-dimensional vector through a two-dimensional dominate/subordinate framework. Each position can be encoded with non-Boolean values so long as the relation between positions is maintained and each value can be translated into its complex representation. The matrices we can derive are as follows

$$x\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + y\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + z\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \delta & \delta - i\delta \\ \delta + i\delta & -\delta \end{pmatrix}$$

The score of each component sums to represent a system whose projection maps the convergence or rotation about that system's determinant towards capturing a position along the real axis, i.e. capturing the actual state of affairs. A rotation in the plane through one of these matrices models degrees of imitation while bargaining for position about the respective axis under investigation. Each component spans an aspect of a three-dimensional plane using complex two-dimensional coordinates. If a functional equivalence across frames of reality obtains between positions relative to frame, then 1 results; if not then 0.

Convergence/divergence can be conceived along *z* such that from *x* we track movement in *y*; from *y* we track movement in *x*; from *z* we track the *x,y* relation along *z*. Using the projection of a frame to organize reality as our basis, running a determinant through a component, respectively, we test i) the convergence between a determinant position projected across different frames as rotation is framed about a position in *x* insofar as *y* or diagonally; ii) convergence between a projected frame across determinant positions as rotation is framed along *z* to *x* or circular about *y*; iii) convergence between different projections, i.e. the extent to which a reality organized along these lines is reproduced across projections for rotation is about z, the projection itself, maintains a relation between frames and positions, a system nested between *x,y*.

9. Discussion

A major change to ART-theory lies in the fact that instead of computing the inner product from a structure that appears self-evident from the record, we test a relation through what is now akin to a 'gate' in the infrastructure organizing a field in which operators negotiate for position for access the resources distributed through those gates, i.e. access to other positions. We see whether the matrix we derive, encoding this process, is a valid interpretation of the record, truer to the initial goals of articulation theory. Individuals may use the identity-markers of others when deciding how to bargain for position, but there is a tradeoff in that the *N* identified with that activity and the one they possess most often cannot be traded. These tags provide the immaterial yet objective asymmetry required to explain how subordinate/dominant relationships are reproduced and maintained.

When these scenarios are mapped, graphing the payoffs shows that some groups organizing around certain markers have higher thresholds for acquiescing their position than others. This translates into a position that can overdetermine the outcome of other's decisions. It is from this point that it's not so hard to extrapolate how these positions are inherited and determine which areas of a domain are accessible or not despite having no value in and of themselves.

How does this affect cultural studies and further Articulation theory's goal? Consider the case of critical fabulation. If the process of articulation can be held in conversation with fabulation in (Black) cultural studies (Hartman 2008) – with fabula, by definition, being the elements with which an explanatory narrative is composed – then we can, through our model, show this process as going through a series of 'logical' and 'chronological' phases related by and through the frameworks, here matrices, organizing transitions between an actor's experience, from one phase or context to another (Hartman 2008). Our findings support the method of critical fabulation as advocated by Saidiya Hartman with Stuart Hall's ART-theory.

As such, critical fabulation in ART-theory is formalized by

$$= \frac{M_1|R\rangle + M_2|R\rangle\ldots}{\sqrt{2}}$$

where each term represents a component licensing what aspects of that stage of the evolution of the subject of study is expressed – by way of a significance, i.e. amplitude, assigned to each – given the conditions in which that framework is applied. Thus, the sum of those components represents which possibilities are articulated given certain conditions, indicating the state of that system at a given point with respect to which alternatives express themselves or not. That output becomes the input for successive applications, narrating a necessary possibility from the archive. From above, we understand that those aspects must exist prior to expression, even if they do not show up in the current state description. The root/route shows how we got our findings through vector addition. Each component is tested through a framing of the record – i.e. past observables updated along this line by current measures – with the resultant vector measuring the plausibility of current or alternative interpretations for how we got to this point. The square of that sum gives us the probability over the distribution of that particular articulation of the subject of study.

An interesting consequence of treating levels of articulation as vectors passing through matrices representing the structures organizing a field – of discourse or operators' affairs – is that when some other M appears between M_H's we can test if appropriation flips power dynamics. M_H flips gates to test inverses, a vector passing through – M_H – Z – M_H – produces a test to see the probability that X emerges, and if – M_H – X – M_H – we find Z. Recall, M_H represents two states that propose a superposition that is not decided until measured – one of the measures survives, the other doesn't. Therefore, the method of fabulation does not produce fiction so long as the narrative vector survives the built-in error correction following the root N rule. Computing alternatives becomes a viable option for study.

10. Conclusion

Treating levels of articulation as vectors passing through phases of organizing structures, once a vector is composed over the sum of its states – assuming that each state has passed through a matrix whose output becomes input towards the composition of the state under investigation – we prepare that vector through a matrix encoding contemporary affairs to test our assumption regarding the life history of that vector derived from the record. This mode of analysis is not to overly complicate the concept of articulation but to show how theory and practice are inextricably linked, regardless of discipline. Since this is what's already occurring during theorization, we've formalized what Hortense Spillers called for in reorienting cultural studies and our role within it. That role being the point of oscillation between competing conceptual claims that provides means for testing those claims' outcomes against what we already know. By forming a guide and method through history that can be shared and corroborated by others, when applied we find similar but not always interchangeable results regarding the emergence of the enabling conditions from which our contemporary state arose. The above represents our attempt to quell onboarding what has been labeled history unquestioningly. If unable to be shared or if posited as having no bounds of application, the method itself must be questioned. Our theories, and the means by which we organize material reality in virtue of what's captured by the matrices we use to understand the world, can be updated, changed, or discarded once better explanations come available, ones that are also useful to others. When taken up by others, it becomes part of the task to see their implication and position in the worlds they go on to construct utilizing these frames.

References

Aaronson, Scott. 2011. "Why Philosophers Should Care About Computational Complexity." arXiv:1108.1791. Last revised August 14, 2011. https://doi.org/10.48550/arXiv.1108.1791.

Bell, Jr. Derrick. 1980. "Brown v. Board of Education and the Interest-Convergence Dilemma." *Harvard Law Review* 93 (3): 518-533.

Blumer, Herbert. 1958. "Race Prejudice as a Sense of Group Position." *The Pacific Sociological Review* 1 (1): 3-7.

Bright, Liam Kofi, Nathan Gabriel, Caolin O'Connor, and Olufemi Taiwo. 2022. "On the Stability of Racial Capitalism." *PhilSci-Archive* [Preprint]. http://philsci-archive.pitt.edu/id/eprint/20664.

DeDeo, Simon. 2016. "Major Transitions in Political Order." arXiv: 1512.03419. Last revised 11 June, 2016. https://doi.org/10.48550/arXiv.1512.03419.

Foucault, Michel. 1966. *Les Mots et les Choses: Une archéologie des sciences humaines* [The Order of Things: An Archaeology of the Human Sciences]. Paris: Éditions Gallimard.

_____. 1975. *Surveiller et punir: Naissance de la prison* [Discipline and Punish]. Paris: Éditions Gallimard.

Gates, Henry Louis Jr. 1988. *The Signifying Monkey: A Theory of Afro-American Literary Criticism.* New York: Oxford UP.

Hall, Stuart. 1973 (1980). "Encoding and Decoding." In *Centre for Contemporary Cultural Studies (Ed.): Culture, Media, Language: Working Papers in Cultural Studies*, edited by Stuart Hall, D. Hobson, A. Lowe, and P. Willis, 128-38. London: Hutchinson.

_____. 1980. "Race, Articulation, and Societies Structured in Dominance." In *Sociological theories: race and colonialism*, 325-330. Paris: UNESCO.

Hartman, Saidiya. 2008. "Venus in Two Acts." *Small Axe* 26 (12): 1-14.

Hempel, Carl Gustav. 1952. *Fundamentals of Concept Formation in Empirical Science*. Chicago: University of Chicago Press.

Mignolo, Walter. 2007. "Introduction: Coloniality of power and de-colonial thinking." *Cultural Studies* 21 (2-3): 155-167.

Mineo, Liz, and Orlando Patterson. 2018. "The Kerner Report on race, 50 years On." *The Harvard Gazette*, March 21, 2018.

Miton, Helena, and Simon DeDeo. 2022. "The Cultural Transmission of Tacit Knowledge." arXiv: 2201.03582. Last revised January 10, 2022. https://doi.org/10.48550/arXiv.2201.03582.

Page, Clarence. 1988. "African American or Black? Pride Comes with Label." *Chicago Tribune*, December 25, 1988.

Peterson II, Victor. 2019. "Raising Pure Hell: A General Theory of Articulation, the Syntax of Structural Overdetermination, and the Sound of Social Movements." PhD diss., King's College London.

_____. Quijano, Anibal. 2000. "Coloniality of Power and Eurocentrism in Latin America." *International Sociology* 15 (2): 215-232.

Scott, La-Toya. 2022. "Black Safe Spaces in Hush to Harbor: Black Culture, Black Literature, and Digital Hashtag." PhD diss., University of Florida. ProQuest (29319355).

Spillers, Hortense. 1994. "The Crisis of the Negro Intellectual: A Post Date." *Boundary 2*, 21 (3): 65-116.

Strawson, Peter F. 1959. *Individuals: An Essay in Descriptive Metaphysics*. Methuen: University Paperbacks.

Taiwo, Olufemi. 2022. *Elite Capture: How the Powerful Took Over Identity Politics (And Everything Else)*. Chicago: Haymarket Books.

Turing, Alan. 1939. "Systems of Logic Based on Ordinals." PhD diss., Princeton University.

Von Eschen, Penny. 2004. *Satchmo Blows Up the World: Jazz Ambassadors Play the Cold War*. Cambridge: Harvard UP.

Wilkerson, Isabel. 1989. "'African-American' Favored by Many of America's Blacks." *New York Times*, January 31, 1989.

Wynter, Sylvia. 2005. "On How We Mistook the Map for the Territory, and Re-Imprisoned Ourselves in Our Unbearable Wrongness of Being, of Désêtre: Black Studies Toward the Human Project." In *Not Only the Master's Tools: African-American Studies in Theory and Practice*, edited by Lewis R. Gordon and Jane Anna Gordon, 107-172. Boulder: Paradigm Publishers.