# Proving Predeterminism, or Why Actuality Is Certainly Actual

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**Abstract:** I define predeterminism as the claim that what is actual is actual with certainty, and provide a proof of it in this paper. Predeterminism solves a major problem: modal realism's probability distributions for selecting the actual world from all the possible worlds, are either arbitrary, because they are not unique, or they do not sum up to one. This problem is solved by replacing modal realism with a set-theoretic plenitude subjected to cosmological natural selection. Essentially, because worlds reproduce with unequal success, and because there are so many of them in the plenitude, worlds outnumber and outweigh each other with infinite factors. An infinitely growing sequence of worlds, or a world life, comes out as the unique champion in this evolutionary competition and is certainly actual. The proof uses the ideas that the probability to be actual for a world is proportional to its size, age, and abundance, and that all the possible worlds are set-theoretically well-ordered by cosmological natural selection.

**Keywords:** set theory, self-locating belief, cosmological natural selection, determinism, necessitarianism, absolute infinite.

## 1. Introduction

Predeterminism is a term that is normally used in religious circles (Newsom 2018). In this paper, it is used to denote the following metaphysical thesis that is proven: actuality is certainly actual. This means that the contingent actuality is uniquely preordained to be the case down to the smallest detail, with 100% certainty. Necessity implies certainty, but not the other way around: a random real number is *certainly* a non-natural number, but not *necessarily* a non-natural number. Likewise, non-actual/actual things are certainly non-actual/actual according to predeterminism, although they remain possible/contingent with respect to modal realism.

Predeterminism follows from a more familiar metaphysical thesis, namely necessitarianism (Mandelkern and Rothschild 2021). The latter is the claim that everything that exists possibly also exists necessarily in actuality (also known as *modal collapse*, or: $\Diamond x \rightarrow \Box x$, where $\Diamond$ means possibility, $\Box$ necessity, and $x$ any world part). Predeterminism and necessitarianism belong to a family of principles that stipulate that some events could not have been any other way. The other members of this family are the principle of sufficient reason (Pruss 2006), fatalism (Bernstein 1992), hard determinism (Pereboom 1995), and soft determinism (Repetti 2010). All these principles, except soft determinism, are often thought to be incompatible with libertarian free will (Palmer 2014).

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Necessitarianism is a well-investigated, but unpopular position. Lewis (1986, 112) stated that modal realism is kaput if all the possible worlds are just parts of actuality. More recently, Mandelkern and Rothschild (2021, 89) could not believe their own technical proof that necessitarianism is true, calling it a puzzle which they leave open. It is my aim in this paper to prove[1] predeterminism, a consequence of necessitarianism, and to provide insight in how it can be the case. The metaphysics behind predeterminism requires what can be known *a priori* from several domains: pure set theory with classes (Fraenkel et al. 1973), self-locating belief (Bostrom 2013), and cosmological natural selection (Smolin 1992). The terminology of modal realism (Lewis 1986) is necessary, although not sufficient, to provide set theory with a metaphysical interpretation. Some extra terminology is introduced to sustain this interpretation.

After a section with background knowledge and assumptions, the proof of predeterminism follows. Then come a discussion and conclusions.

## 2. Background Knowledge and Assumptions

### 2.1 Modal realism

According to modal realism, all the possible worlds exist. They are causally and spatiotemporally isolated (henceforth *isolated*), and the union of all the worlds is all of reality and is called *the plenitude*[2] (PL). Something exists possibly if it exists in some possible worlds and necessarily if it exists in every possible world. Something is contingent if it is possible, but not necessary.

Two worlds or world parts[3] are *duplicates* if and only if (henceforth iff) all their intrinsic properties are the same. Every world is a *spacetime* and I will define spacetime in Section 2.2 such that also the plenitude is a spacetime (although not a world). Let x, y, and z be symbols reserved for worlds or spacetimes. Then the *abundance* of x in y is the number of duplicates x has in y.

The *actual* world is our world. This means that the actuality of a world is indexical, just like 'here' and 'now.' According to modal realism, the actual world is non-plenitudinous with certainty.[4] This interpretation of modal realism has a major problem: any distribution of uncertain probabilities (different from zero and one) for selecting the actual world from all the possible worlds (Nilsson 1986, 72), is arbitrary and/or does not sum up to one.

These uncertain-valued probability distributions typically become smaller at an exponential rate, because they have to sum up to one. Consider, for example, the exponential distribution in which world *n* gets probability $1/2^n$, with *n* any

[1] Blondé's (2015, 148-150) paper paved the way for this proof, however, with serious flaws. For example, even though it uses set theory, it does not use the notion of well-orderedness.

[2] Lewis also often uses the term 'logical space.'

[3] In this paper, a world x is part of a world y iff x has a duplicate that is part of y. I will, therefore, speak of worlds instead of world parts.

[4] See Lewis' (1986, 101) 'All worlds in one?'

natural number. This probability distribution indeed sums up to one. However, there are infinitely many of this sort of probability distributions that sum up to one. Therefore, choosing one of these probability distributions is arbitrary. This problem will be solved by adding a non-arbitrary selection principle, namely cosmological natural selection, and the conclusion that cosmological natural selection in a plenitude without isolation results in a probability distribution that has no other values than zero and one.

There is a second problem with the interpretation that the actual world is non-plenitudinous with certainty. This interpretation goes against Bostrom's (2013) Self-Indication Assumption, which is the following proposition:

> Self-Indication Assumption: Given the fact that you exist, you should (other things equal) favor hypotheses according to which many observers exist over hypotheses on which few observers exist (Bostrom 2013, 66).

Indeed, it is more likely to be born in a world with many observers as compared to a world with few observers. However, Bostrom is himself not a fan of the Self-Indication Assumption, because it predicts that there is an infinite number of observers in the actual world. He reasons that an infinite number of observers is not an observed fact, and that *a posteriori* evidence has priority over *a priori* reasoning. I argue that it is impossible to conclude from *a posteriori* evidence that there is a finite number of observers in the actual world. Since gathering *a posteriori* evidence is limited by our measuring instruments, we have to use *a priori* principles like the Self-Indication Assumption to make conclusions about those parts of the actual world that we cannot observe. Given that a plenitudinous world maximizes the number of observers, and that the actual world is non-plenitudinous according to modal realism, it follows that modal realism goes against the Self-Indication Assumption. This problem is solved if the claim that the plenitude is not actual is abandoned.

### 2.2 Set theory

In pure, well-founded set theory, sets are iteratively defined starting from the empty set toward sets that contain hereditarily only sets as elements (pureness), and such that there are no infinitely long 'has as element' chains (well-foundedness). Set theory can formalize actual infinities, also called *transfinite* numbers, which are the extensions of the natural numbers beyond ω, the smallest transfinite number.

A fraction of two numbers is zero if the denominator is at least ω times greater than the numerator. This enables the usage of classical, Archimedean probabilities, rather than non-zero probabilities for infinitely unlikely events (Benci et al. 2018). The distinction between impossible/necessary and possible with probability zero/one is not needed for proving predeterminism. It suffices to show that either the necessity or the certainty of actuality is the case.

All sets are classes, but not every class is a set. For example, the class of all sets (also called the universal class) is a proper class, which means that it is itself not a member[5] of a class, and, therefore, not a set. This avoids the inconsistencies known from naive set theory. Proper classes have a size that is equal to Ω, which is an Absolute Infinite akin to Cantor's Absolute Infinite (Jané 1995, 375). Cantor defined his Absolute Infinite as an inconsistent cardinal or ordinal that exceeds every cardinal or ordinal. However, I use the definition that Ω is a unique absolute number that is neither a cardinal nor an ordinal number, and that uniquely exceeds every conceivable cardinal and ordinal number. I will therefore call cardinals and ordinals *exceedable* numbers. This distinction between numbers, exceedable numbers, and the absolute number compares to the distinction between classes, classes that are sets, and proper classes. These distinctions between types of numbers are necessary to avoid Cantor's naive self-referential inconsistency.

Set theories with classes use the Axiom of Limitation of Size (Fraenkel et al. 1973, 119). This axiom warrants that Ω-large recombinations[6] of classes cannot exceed the size Ω. Because of this, Ω plays an important role in the proof of predeterminism.

This proof also requires the notions of a total-ordered relation, a well-founded relation, and a well-ordered relation. A relation $R$ on a set of elements is a total-ordered relation iff $R$ is reflexive (x$R$x), transitive (if x$R$y and y$R$z, then x$R$z), antisymmetric (if x$R$y and y$R$x, then x = y), and total (x$R$y or y$R$x, or both). A relation $R$ on a set S of elements is well-founded[7] iff there are no infinitely long descending chains along $R$,[8] for chains that have distinct neighbors. This is equivalent to saying that there are no subsets of S that have no minimal element according to the ordering imposed by $R$. For a minimal element x there are no elements y such that y$R$x, with x and y distinct. Mind that a set can have multiple *minimal* elements, but only one least (or *minimum*) element. A relation $R$ on a set S of elements is well-ordered iff every subset $S_i$ of S has a (unique) minimum element x. In symbolic language: $\forall\, S_i\colon \exists!\, x, \forall\, y \in S_i\colon x\ R\ y$. Moreover, $R$ is well-ordered iff $R$ is total-ordered and well-founded.

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[5] While sets have elements, classes have members.

[6] Lewis (1986, 103) hoped for a natural break to the plenitude that would 1) avoid unlimited recombination, 2) not disqualify worlds that are possible, and 3) not be intolerably *ad hoc* as a boundary. I contend that only Ω can be such a break.

[7] Mind the difference with a strictly well-founded relation, which is well-founded and irreflexive.

[8] This has a consequence that can be expressed in terms of *limit elements* (or limit ordinals, Jech 1997, 20). Let C be an infinite chain of elements that is ordered by a relation $R$ between elements. Then an element z is a limit element of C iff for every element x in C that is ordered earlier than z, there exists an element y in C such that x$R$y$R$z. Well-foundedness implies that each element is only a finite number of has-as-predecessor steps away from any limit element, so in the descending direction (toward the earliest element). Ascending chains can have any transfinite number of has-as-successor steps.

At this point, we can formulate the first assumption for proving predeterminism:

> (I) Set-Theoretic Plenitude: There is a one-to-one correspondence between pure, well-founded sets and worlds.

The defense of Set-Theoretic Plenitude is that there is a one-to-one correspondence between pure, well-founded sets and exceedable numbers, between exceedable numbers and ordinal machines (the transfinite extensions of computers) (Koepke and Seyfferth 2009), and between ordinal machines and worlds. The first correspondence is well understood, given that exceedable numbers are defined via pure, well-founded sets. With respect to the second correspondence, for each exceedable α there is an ordinal machine that has a memory that consists of α discrete cells and α time instants that are available to finish the calculation. The third correspondence is evident from the fact that for every world there is an ordinal machine that simulates that world, and that for every ordinal machine there is a world (if necessary with a transfinite size) that has this ordinal machine as proper part. In this way, worlds are explanatorily derived from necessarily existing abstract entities. After all, intelligent beings that are simulated by an abstract computer program, cannot conduct any experiment to find out whether they are abstract or physical.

Sets, exceedable numbers, and ordinal machines have a discrete nature. Because of the one-to-one correspondence, also worlds are spatiotemporally discrete: they consist of an exceedable number of point-instants. This makes them compatible with set theory, because it warrants they can have well-ordered properties (Schust 2019).

Set-Theoretic Plenitude lays the foundation for further correspondences between physical entities and pure set theory with classes: worlds correspond to sets and exceedable numbers (hence non-Ω-large entities), the plenitude to the universal class and Ω, spacetimes to classes, and the *minimum world* (a single point-instant) to the empty set.[9] The relation *is proper part of* corresponds to the transitive closure of the *is element of* relation. So a world x is proper part of a world y iff x as set is an element of an element, etc. of y as set.

I introduce some extra terminology that is needed for the proof of predeterminism: the size-age of a world, the total size-age of a world x in a world y, the total size-age ratio of a world x in a world y (from which the 'materializes P% of' relation is derived), partonomic compatibility, partonomic completeness, initial segment, and a world life.

The *size-age* (size multiplied by age or longevity) of a world is determined by the number of point-instants of which it consists. The *total size-age* of a world x in a world y, or TotalSizeAge(x, y), is the combined size-age of all the duplicates of x that are part of y. The *total size-age ratio* of a world x in a world y, or

[9] In order to use familiar terminology, I will speak of 'world' as much as possible. Technically, 'spacetime' is often more appropriate, even when referring to worlds.

TotalSizeAgeRatio(x, y), is the total size-age of x in y, divided by the size-age of y. We have $0 \leqslant$ TotalSizeAgeRatio(x, y) $\leqslant 1$. If TotalSizeAgeRatio(x, y) = P%, then x *materializes P% of* y. With that, we can express the following assumption:

> (II) Worldly Self-Indication: The probability that a world x is actual is TotalSizeAgeRatio(x, Plenitude).

Worldly Self-Indication assumes that a proverbial arrow is randomly shot in the size-age of the plenitude, thereby giving each elementary bit of size-age (or each minimum world) in the plenitude an equal probability of being hit. A necessary and sufficient condition for a world to be actual, is that a part of it is hit. Consequently, both the minimum world and the plenitude are actual with certainty. The actuality of the worlds in between these two extremes is what predeterminism is about.

Worldly Self-Indication favors the actuality of worlds with a great abundance, a great size, and a great age. That is in accordance with the classical Self-Indication Assumption. While the Self-Indication Assumption is about the probability to be born as a specific observer, Worldly Self-Indication is about the probability that a specific world is actual.

Let a pair of worlds x and y be *partonomically compatible* iff x is part of y, or y is part of x. Otherwise x and y are partonomically incompatible. Then a *world life* (henceforth *life*) in a world x is any *partonomically complete* sequence of pairwise partonomically compatible *initial segments* of x. An initial segment of a world x is a world that contains everything in x until a given time instant in x. Let S be a sequence of partonomically compatible initial segments of a world w and let S be ordered by the *is proper part of* relation. Then S is partonomically complete iff 1) S contains the minimum world and w, 2) there exists no world z for any pair of adjacent worlds x and y in S, such that x is proper part of z and z is proper part of y, and 3) all the limit worlds of S are in S. A life in a world x is therefore a steadily growing world in x that starts with a minimum world and ends with x.

## 2.3 Cosmological natural selection

Cosmological natural selection (Smolin 1992) is the theory that Darwinian natural selection is universally applicable. It follows from the idea that Darwinian natural selection can – in theory – be derived *a priori*, so via logical thinking alone. Indeed, worlds that reproduce[10] well will be more abundant than worlds that reproduce poorly, which is the essence of natural selection. With that, I propose Cosmological Natural Selection (assumption III) and two theorems that are consequences of it:

> (III) Cosmological Natural Selection: Darwinian natural selection is applicable in a random limit to the plenitude.

[10] Smolin proposed that big bang universes can self-reproduce via black holes.

Cosmological natural selection in a plenitude is thereby held to be a process that sets up an evolutionary competition between every pair of worlds in the plenitude. An assumption or a relation holds *in a random limit to the plenitude* if it holds in all the worlds that are one of a randomly chosen sequence of partonomically compatible worlds $z_\alpha$ that diverges to the plenitude, starting from a sufficiently great α-index. A randomly chosen sequence is required, because in a plenitude there are necessarily infinitely unlikely sequences of worlds $z_\alpha^*$ in which an assumption like cosmological natural selection is counteracted. For example, a world that reproduces poorly can be given an arbitrarily great abundance in a specially chosen sequence of $z_\alpha^*$'s.

A random selection of the $z_\alpha$'s starts with the selection of a world $z_1$, by randomly selecting a world duplicate from the class of all (equiprobable) world duplicates. Then a world $z_2$ is selected via a random selection from the class of all world duplicates that have $z_1$ as proper part, followed by a similar selection of a $z_3$ that has $z_2$ as proper part, etc. (selecting limit worlds when appropriate), until the plenitude is selected. If predeterminism is correct, however, the assumption of randomness must lead to its own contradiction. Indeed, the selection method above selects the partonomically complete, certainly actual life $L_A$ with certainty, because $L_A$'s initial segments all materialize 100% of the plenitude.[11] Therefore, it suffices to assume that Darwinian natural selection is applicable in all sufficiently great $z_\alpha$ that materialize 100% of the plenitude.

This brings us to the consequences of Cosmological Natural Selection. Let the relation *evolutionarily precedes* be defined such that x evolutionarily precedes y iff, in a random limit to the plenitude, the reproduction of a (duplicate of) y goes together with the reproduction of a (duplicate of) x, rather than the other way around. Reproduction can happen via any process and is not limited to self-reproduction. Even though evolutionary precedence is about an abundance relation between x and y in a random limit to the plenitude, it can, in many cases, be interpreted in biological terms. For example, bacteria evolutionarily precede eukaryotes because eukaryotes turned free-living bacteria into organelles. Therefore, eukaryotes can (technically[12]) not reproduce themselves without also reproducing bacteria, while many bacteria can reproduce without necessitating the reproduction of a eukaryote. Another example is that lions require the reproduction of prey for their own reproduction, while prey can reproduce independently.

The first consequence of Cosmological Natural Selection is Total-Orderedness Theorem:

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[11] Blondé (2015, 150) constructs the certainly actual part of the plenitude as a 'life' $L_A^*$ with Ω-large initial segments $i_\alpha^*$. This $L_A^*$ contains all the world duplicates generated in the plenitude by the initial segments $i_\alpha$ of $L_A$. Recursively, we have $i_1^* = i_1$ and $i_{\alpha+1}^* = (\Omega \times i_\alpha^*) + i_{\alpha+1}$, with $i_1$ the minimum world. This construction makes it clear how all the $i_\alpha$'s can materialize 100% of the plenitude simultaneously.

[12] In fact, the organelles are no longer called bacteria.

Total-Orderedness Theorem: The relation *evolutionarily precedes or is equal to* is total-ordered on the plenitude.

*Proof*. A relation is total-ordered iff it is 1) reflexive, 2) transitive, 3) antisymmetric, and 4) total. 1) The relation *evolutionarily precedes or is equal to* is explicitly made reflexive via the *or is equal to* part. 2) Precedence, having a greater reproductive ability in a random limit to the plenitude, and equality are all transitive relations. Therefore *evolutionarily precedes or is equal to* is also transitive. 3) The *rather than the other way around* in the definition of evolutionary precedence explicitly makes this relation antisymmetric. 4) Totality follows from the idea that even the smallest difference between two worlds x and y results in a difference in reproductive ability of x and y within a sufficiently large world $z_\alpha$. This difference in reproductive ability is hard to deny for big differences. Duplicates of four-legged dogs clearly reproduce or are reproduced more abundantly than duplicates of one-legged wolves. However, what holds for big differences – ultimately, in a random limit to the plenitude – holds for every difference, no matter how small it is. Therefore, a difference in internal make-up results in a difference in reproductive ability in a random limit to the plenitude, and this in turn in an evolutionary precedence for any pair of distinct worlds x and y. □

The second consequence of Cosmological Natural Selection is Well-Foundedness Theorem. Its proof requires the notions of *regular* and *irregular evolutionary chains*. In a regular evolutionary chain, the earlier worlds evolutionarily precede the later worlds[13] and descending backward toward the earliest element always results in a finite number of steps. Irregular evolutionary chains, on the other hand, are infinitely long chains in which the earlier worlds are evolutionarily preceded by the later worlds, instead of the other way around.

Also this notion of evolutionary chains is known from biology and physics: particles, atoms, molecules, biomolecules, cells, eukaryotes, plants, and animals, form an example of such a chain. According to Set-Theoretic Plenitude and Cosmological Natural Selection, evolutionary chains can have Ω levels: there are 4D worlds that require 3D animals as a prior, 5D worlds that require 4D worlds as a prior, etc., all the way to the ΩD plenitude.[14]

I will prove irregular evolutionary chains do not exist via the following theorem:

[13] The later worlds are then evolutionarily dependent on (or require as a prior) the earlier worlds in the chains, while the earlier worlds are evolutionarily conserved (unalterable) (Ferrière et al. 2004; Blondé 2016).

[14] Because the observable world is unalterable due to evolutionary conservation, we can only observe the earlier elements of the chain: from particles to 3D animals (Blondé 2016).

Well-Foundedness Theorem: The relation *evolutionarily precedes or is equal to*[15] is well-founded on the plenitude.

*Proof*. Given Set-Theoretic Plenitude, there exists a minimum world (a point-instant) in the plenitude that has the greatest abundance of all worlds. Consequently, 1) the minimum world evolutionarily precedes every other world. 2) In a random limit to the plenitude, every world, no matter how large, chaotic, or malevolent, eventually becomes useful in some reproduction cycle (if needed as a carrier of genetic information or an ornament of some advanced life form). From 1) and 2) it follows that 3) in a random limit to the plenitude, every world x eventually becomes an element of one or more regular evolutionary chains that start from the minimum world. (Such regular chains can consist of the minimum world, evolutionarily earlier worlds in the reproduction cycle of x, and x itself.)

According to the definitions, the relation *evolutionarily precedes or is equal to* is well-founded iff no irregular evolutionary chain exists. Therefore, as a proof from contradiction, I propose the hypothesis that an irregular evolutionary chain exists. However, the earlier worlds in such a chain have arbitrarily many different additional regular evolutionary chains along which they can come into existence and reproduce abundantly, as compared to an arbitrarily late world in the irregular chain.[16]

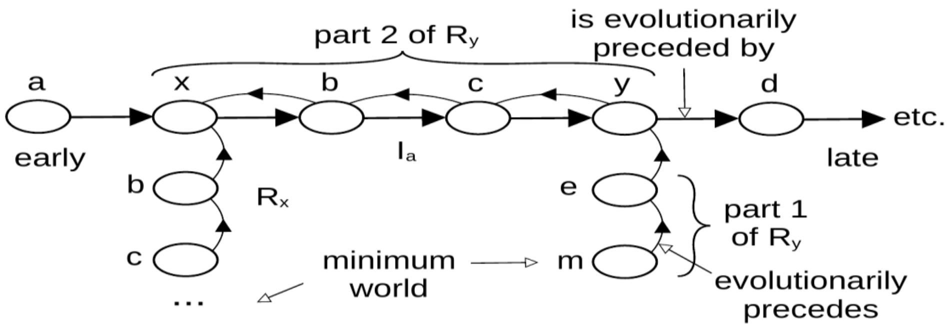


Figure 1: Two regular evolutionary chains ($R_x$ and $R_y$) and one irregular chain ($I_a$). $R_x$ = (m, …, c, b, x), $R_y$ = (m, e, y, c, b, x), and $I_a$ = (a, x, b, c, y, d, …). Because the descending chain (b, c, …, m) in $R_x$ must be finite, there are infinitely many additional regular chains like $R_y$ through which the early world x can come into existence. E.g. $R_d$ = (m, d, y, c, b, x).

Indeed, because of 3), all the later worlds y in the infinite irregular chain create additional regular evolutionary chains for the earlier worlds x (See Figure

[15] Well-founded relations may or may not be reflexive. Therefore, *evolutionarily precedes or is equal to* is well-founded iff *evolutionarily precedes* is well-founded.

[16] For the case that the infinite irregular chain consists of a finite number of worlds that are short circuited on themselves, it suffices to note that the relation *evolutionarily precedes* is transitive and irreflexive: worlds cannot evolutionarily precede themselves.

1). These regular chains have two parts: part 1 from the minimum world to y outside of the irregular chain, and part 2 from y to x along the inverse direction of the irregular chain. Some of these regular chains $R_y$ can already exist for an earlier world x as a chain $R_x$. This happens when a section of part 1 of $R_x$ that touches x coincides with the section of $R_y$ that comes after x. Nevertheless, a different regular chain $R_y$ is certain to follow for some later world y, because a regular chain $R_x$ can map only a finite number of its x-touching-worlds-in-part-1-of-$R_x$ to the worlds that come after x in the irregular chain.[17] Otherwise $R_x$ would not be regular but irregular. This different $R_y$, with y coming after x plus its finite mapping to part 2 of $R_y$, is an additional regular evolutionary chain.

4) Any additional evolutionary chains contribute to the abundances of the early worlds, but not to those of the late worlds. Moreover, 5) if the arbitrarily late worlds are formed via arbitrarily long regular evolutionary chains, then these regular chains contribute less to the abundances of the late worlds as compared to the shorter regular chains for the early worlds, because in regular chains the early worlds have, according to definition, the greatest abundances. As a result of 4) and 5), the later worlds in an irregular chain cannot have the greater abundances in a random limit to the plenitude and, therefore, cannot evolutionarily precede the earlier worlds. This contradicts the hypothesis. Therefore, the relations *evolutionarily precedes* and *evolutionarily precedes or is equal to*, are well-founded on the plenitude. □

With the assumptions and their consequences in this section we can start with the proof of predeterminism. This happens in the next section.

## 3. Proving Predeterminism

In the previous section, two theorems about the *evolutionarily precedes or is equal to* relation were proven: Total-Orderedness Theorem, and Well-Foundedness Theorem. In this section, four more theorems about two more relations are proven: First Equality Theorem, Second Equality Theorem, Well-Ordering Theorem, and Certain Life Theorem, about the relations *Ω-outnumbers or is equal to* and *part-or-outweighs or is equal to*. Certain Life Theorem claims that a unique, partonomically complete life $L_A$ is certainly actual, which proves predeterminism.

Let the relation *Ω-outnumbers* be defined such that x Ω-outnumbers y iff the abundance ratio of x and y in z diverges exclusively[18] to Ω for z going in a random limit to the plenitude. With that, we can express First Equality Theorem:

> First Equality Theorem: The relations *evolutionarily precedes or is equal to* and *Ω-outnumbers or is equal to* are equal: a world x evolutionarily precedes or is equal to a world y iff x Ω-outnumbers or is equal to y.

[17] This mapping should exclude the minimum world, which can never be in the middle of an evolutionary chain.

[18] This excludes the possibility that the abundance ratio keeps both exceeding every ratio α and dropping below every ratio 1/α, as z goes in the limit to the plenitude.

*Proof*. Let us postpone the issue of reflexivity and assume that x and y are distinct. Then the proof consists of the following two steps: Step 1) if x Ω-outnumbers y then x evolutionarily precedes y, and Step 2) if x evolutionarily precedes y then x Ω-outnumbers y.

Step 1) If x Ω-outnumbers y, then (trivially) the reproduction of a (duplicate of) y more often goes together with the reproduction of a (duplicate of) x in a random limit to the plenitude, than the other way around. According to the definition of the relation *evolutionarily precedes*, x then evolutionarily precedes y.

Step 2) If x evolutionarily precedes y, then, 2a) if at all either x Ω-outnumbers y, or y Ω-outnumbers x, then x Ω-outnumbers y. Indeed, in worlds $z_\alpha$ that go in a random limit to the plenitude, a reproduction of y goes together with a reproduction of x, rather than the other way around. Therefore the abundance of x in $z_\alpha$ must rather be greater than the abundance of y in $z_\alpha$ in a random limit to the plenitude, such that y can never Ω-outnumber x. 2b) For worlds $z_\alpha$ that go in a random limit to the plenitude, the number of (non-duplicate[19]) worlds in $z_\alpha$ diverges to Ω. If a pair of worlds x and y would in all circumstances reproduce together, such that χ new duplicates of x come with γ new duplicates of y, then the abundance ratio would not diverge, but remain χ/γ. However, this is not the case for most x and y. Therefore, also the abundance ratios between most pairs of worlds in $z_\alpha$ diverge to either Ω or 1/Ω.

As a result of 2a) and 2b), for most pairs of x and y in which x evolutionarily precedes y, x Ω-outnumbers y. Now I argue that, in a random limit to the plenitude, x Ω-outnumbers y for *every* pair of worlds x and y in which x evolutionarily precedes y.

Let us consider a proof by contradiction by assuming that there exists a special pair of worlds $x^*$ and $y^*$, where $x^*$ evolutionarily precedes $y^*$, and such that their abundance ratios stay below or keep dropping below a certain ratio β > 1/Ω, in $z_\alpha$'s that go in a random limit to the plenitude. The larger $z_\alpha$ becomes, the more coincidental and special this relation between $x^*$ and $y^*$ becomes. However, using the assumptions of Set-Theoretic Plenitude, Cosmological Natural Selection, and the randomness of the sequence of worlds $z_\alpha$, there is nothing that indicates a special relation between any pair of worlds. Even though such a special relation can exist in some particular world $z_\alpha$, there are always Ω randomly chosen worlds left that will come in an evolutionary competition with $z_\alpha$ and that jointly contain Ω $x^*$ and $y^*$ duplicates. On the other hand, there can only be an exceedable number of $x^*$ and $y^*$ duplicates in the union of all the worlds that are not larger than $z_\alpha$. Therefore, the probability that $x^*$ and $y^*$ do not diverge is 1/Ω. Because there are only an exceedable number of possible pairs of $x^*$ and $y^*$ below any exceedable threshold, it follows that such an $x^*$ and $y^*$ do not exist and that x Ω-outnumbers y.

[19] Obviously, the number of world duplicates diverges to Ω. However, a world represents a type of world duplicate, and also the number of such types diverges to Ω.

The combination of Step 1 and Step 2 proves the equality of the relations *evolutionarily precedes or is equal to* and *Ω-outnumbers or is equal to* for distinct x and y. Because both these relations are reflexive, this proves their equality. □

Next, I introduce the relations *Ω-outweighs* and *part-or-outweighs*. Let us call the division of the total size-age of x in z by the total size-age of y in z, the outweigh-ratio of x and y in z:

$$\text{outweigh-ratio of x and y in z} = \frac{TotalSizeAge(x,z)}{TotalSizeAge(y,z)}$$

The outweigh-ratio of x and y in z indicates which fraction of z is materialized by x as compared to y. If z has a transfinite size-age, the outweigh-ratios can also become transfinite. Let us say that x Ω-outweighs y iff the outweigh-ratio of x and y in z diverges exclusively to Ω for z going randomly in the limit to the plenitude. Then the relation *part-or-outweighs* reads as: *is proper part of or Ω-outweighs*. The following formula recapitulates this:

$$x\ PartOrOutweighs\ y$$

$$\leftrightarrow$$

$$(x\ ProperPartOf\ y)\ \vee \left(\lim_{z \to \mathrm{PL}} \frac{TotalSizeAge(x,z)}{TotalSizeAge(y,z)} = \Omega\right)$$

Having introduced the *part-or-outweighs* relation, we can express Second Equality Theorem. Its proof is more familiar, in the sense that it does not depend on Cosmological Natural Selection.

> Second Equality Theorem: The relations *evolutionarily precedes or is equal to* and *part-or-outweighs or is equal to* are equal: a world x evolutionarily precedes or is equal to a world y iff x part-or-outweighs or is equal to y.[20]

*Proof*. Let us again postpone the issue of reflexivity and assume that x and y are distinct. Then the proof consists of the following two steps: Step 1) if x part-or-outweighs y then x evolutionarily precedes y, and Step 2) if x evolutionarily precedes y then x part-or-outweighs y.

Step 1) If x part-or-outweighs y then either 1a) x is proper part of y or 1b) x Ω-outweighs y. 1a) If x is proper part of y then x always evolutionarily precedes y because a reproduction of y necessitates a reproduction of x, while not the other way around. 1b) If x Ω-outweighs y, then x must also Ω-outnumber y, because the size-ages of x and y have exceedable values. (If x would only α-outnumber y, with α some exceedable value, then x would also only β-outweigh y, with β some other exceedable value.) Because of First Equality Theorem, x then evolutionarily precedes y. In either case, 1a) or 1b), x evolutionarily precedes y.

[20] Consequently, the three relations evolutionarily precedes or is equal to, Ω-outnumbers or is equal to, and part-or-outweighs or is equal to are all equal.

Step 2) If x evolutionarily precedes y then, according to First Equality Theorem, x Ω-outnumbers y. Now, if 2a) x is not proper part of y,[21] then we have three facts: Fact 1) x materializes 0% of y. Fact 2) y materializes 0% of x. (Otherwise y would be a proper part of x, which would imply that y evolutionarily precedes x. This is impossible, because evolutionary precedence is antisymmetric.) Fact 3) the size-ages of x and y have exceedable values.

With these three facts, we can calculate the outweigh-ratio of x and y in $z_\alpha$, with $z_\alpha$ going in a random limit to the plenitude. Indeed, because of Fact 1) and 2) the x and y duplicates occupy distinct regions of the plenitude. This calculated ratio is an exceedable value (the division of the size-age of x by the size-age of y, and using Fact 3)) multiplied by the abundance of x in the plenitude, divided by the abundance of y in the plenitude. Because x Ω-outnumbers y, the division of the last two factors is Ω, such that x also Ω-outweighs y. If x Ω-outweighs y, then x part-or-outweighs y, according to the definition of *part-or-outweighs*. If 2b) x is proper part of y, then also x part-or-outweighs y, according to this definition.

The combination of 2a) and 2b) proves Step 2. The combination of Step 1 and Step 2 proves the equality of the relations *evolutionarily precedes or is equal to* and *part-or-outweighs or is equal to* for distinct x and y. Because both these relations are reflexive, this proves their equality. □

Now we can prove the following theorem:

> Well-Ordering Theorem: The relation *part-or-outweighs or is equal to* is well-ordered on the plenitude.

*Proof*. According to Total-Orderedness Theorem and Well-Foundedness Theorem, the relation *evolutionarily precedes or is equal to* is total-ordered and well-founded on the plenitude. Such a relation is well-ordered on the plenitude. Because of Second Equality Theorem, also the relation *part-or-outweighs or is equal to* is well-ordered on the plenitude. □

Let us now call a world that part-or-outweighs all the other worlds in a set of worlds S, a part-or-outweigh champion of S. From Well-Ordering Theorem it follows that in every set of worlds, there is a unique part-or-outweigh champion. With that, Certain Life Theorem can be proven:

Certain Life Theorem: A unique life $L_A$ in the plenitude can be constructed, such that its partonomically complete sequence of initial segments are certainly actual.

[21] Proper parts do not Ω-outweigh all their wholes. For example, even though the minimum world Ω-outnumbers the plenitude, it does not Ω-outweigh it.

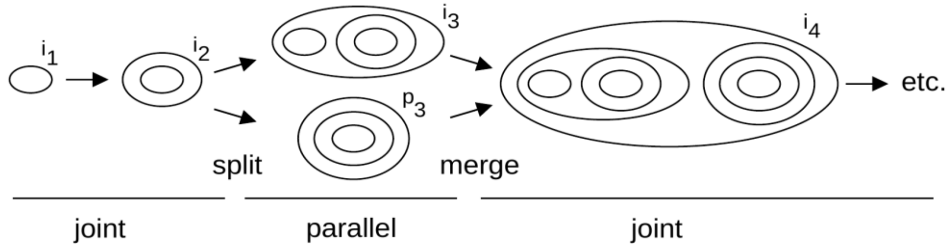


Figure 2: The two phases of two lives that are compared, via their translations to pure sets. According to predeterminism, either the worlds in the top or the bottom life in the parallel phase (either i3 or p3 here) are absolutely infinitely more likely to be actual than those in the other life.

*Proof*. We begin with the minimum world as the first world $i_1$ that is selected as an initial segment of the constructed life $L_A$. For finding the next initial segment $i_{\alpha+1}$, we construct the class $W_\alpha$ of all the worlds that have $i_\alpha$ as proper part. Because of Well-Ordering Theorem, a unique part-or-outweigh champion in $W_\alpha$ can always be found. This part-or-outweigh champion is selected as the new initial segment $i_{\alpha+1}$ of $L_A$. A limit world is selected when all the proper parts of the limit world are a proper part of an initial segment that has already been selected. This iteration continues until the plenitude is selected. With that, we have constructed $L_A$: a sequence of initial segments $i_\alpha$ that starts with the minimum world and ends with the plenitude.

I will now prove that $L_A$ is the certainly actual life by proving, first, that the constructed sequence is partonomically complete and then, second, that the initial segments in the sequence are certainly actual. First, for the claim about partonomic completeness, I propose a proof by contradiction by assuming that there is a *j* such that $i_\alpha$ is proper part of *j* and *j* is proper part of $i_{\alpha+1}$, for some α. If this *j* exists, the constructed sequence is not partonomically complete. However, if this *j* exists, it would be in the class $W_\alpha$, because *j* has $i_\alpha$ as proper part. Moreover, this *j* would part-or-outweigh $i_{\alpha+1}$, because *j* is proper part of $i_{\alpha+1}$. This is not possible by construction, given that $i_{\alpha+1}$ is the part-or-outweigh champion in $W_\alpha$. Such a *j* can, therefore, not exist. Because the minimum world, all the limit worlds, and the plenitude are also included in the sequence, this shows that the constructed sequence is partonomically complete.

Second, I prove that the initial segments of $L_A$ are certainly actual. For each initial segment $i_\alpha$ of $L_A$, we compare $i_{\alpha+1}$ with the parallel initial segments $p_{\alpha+1}$ of lives that split away from $L_A$ at index α, such that the $p_{\alpha+1}$'s do have $i_\alpha$ as proper part, but not $i_{\alpha+1}$ (see also the parallel phase in Figure 2). These $p_{\alpha+1}$'s together with $i_{\alpha+1}$ are precisely the worlds in a reduced $W_\alpha^*$: all the worlds in $W_\alpha$ that have $i_{\alpha+1}$ as proper part are removed from $W_\alpha^*$. Within $W_\alpha^*$, the part-or-outweigh champion $i_{\alpha+1}$ not only part-or-outweighs all the $p_{\alpha+1}$'s, it also Ω-outweighs all the $p_{\alpha+1}$'s. Indeed, according to the definition of the *part-or-outweighs* relation, a world x that part-or-outweighs a world y, also Ω-outweighs y if x is not a proper part of y.

Since $i_{\alpha+1}$ Ω-outweighs all the $p_{\alpha+1}$'s, it also Ω-outweighs all the duplicates of $i_\alpha$ that are not a proper part of an $i_{\alpha+1}$-duplicate, but a proper part of one of the duplicates of one of the $p_{\alpha+1}$'s. This must be so because all these $i_\alpha$-duplicates-in-$p_{\alpha+1}$-duplicates together do not materialize more of the plenitude than all the $p_{\alpha+1}$-duplicates together,[22] which is 0%. This means that if $i_\alpha$ materializes 100% of the plenitude, that also $i_{\alpha+1}$ does so, because 100% of the $i_\alpha$ duplicates are found in an $i_{\alpha+1}$-duplicate.

Now, given that $i_1$ (the minimum world) is the fundamental unit of size-age, it materializes 100% of the plenitude. This proves, via transfinite induction (Jech 1997, 21), that all the $i_\alpha$ 's materialize 100% of the plenitude, or TotalSizeAgeRatio( $i_\alpha$ , Plenitude) = 1, for all α. This makes all the $i_\alpha$ 's of $L_A$ (including the plenitude) certainly actual. We have thus constructed a unique life in the plenitude whose partonomically complete sequence of initial segments are certainly actual and ordered by the *is proper part of* relation. This proves predeterminism. □

## 4. Discussion

### 4.1 Randomizing probabilities

Given predeterminism, there are no probabilities different from zero and one that a world is actual. This is a virtue of predeterminism, because it avoids the need to randomize the probability distribution over all the possible worlds in the plenitude. A randomizer – one that does not always return the same outcome – cannot be implemented in a computer program without making use of an input (or a seed) derived from an external reality (Koza 1991). However, with respect to the whole plenitude, there is no external reality. A randomizer that uses internal facts, either hardcoded in the randomizer or coming from one or more of the possible worlds, will always return the same outcome, which results in predeterminism again.

### 4.2 Solipsistic predeterminism

As I start from modal realism in this paper, and because modal realism is about worlds instead of conscious experiences or brain structures, I have restricted the definition of predeterminism to be about the actuality of worlds. Nevertheless, Blondé (2015) proposes a more controversial variety of predeterminism that could be called *solipsistic predeterminism*, which adds the claim that the certainly

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[22] Because *part-or-outweighs* is well-founded on the plenitude, it can be generalized from worlds (sets) to spacetimes (classes) (Jech 1997, 67). Therefore, the spacetime-union of all the $p_{\alpha+1}$-duplicates is part-or-outweighed and Ω-outweighed by $i_{\alpha+1}$, such that it materializes no more than 0% of the plenitude.

actual life is the evolving consciousness of a unique observer. This is a form of solipsism in which other observers do exist, however, the brain structures that produce their conscious experiences have total size-ages that are infinitely smaller than those of the solipsistic experiences.

As a cosmological model, Blondé (2016, 32) proposes a Russian nesting doll of dimensionally smaller universes in dimensionally larger universes. Complex life in higher dimensions becomes evolutionarily dependent on less complex life in lower dimensions. In order to reproduce themselves, more complex agents efficiently reproduce, simulate, back up, and restore less complex agents, while not interfering out of respect for their evolutionarily conserved reproduction plan. In this way, the density of intelligent consciousness-generating brain or CPU matter always increases in higher dimensions. In the limit, such brain or CPU matter materializes 100% of the plenitude and simulates us (with unequal resources) as we progress through increasingly many spatial dimensions.

Solipsism may not appear an attractive worldview. However, as the density of brain matter increases in higher-dimensional layers of reality, the probability increases that brains – and the consciousnesses they generate – merge (for example, by connecting neurons). In this way, eventually everybody will become the certainly actual consciousness, by merging with it in a plenitude-wide brain.

## 5. Conclusions

According to the account of predeterminism in this paper, actuality is the unique outcome of an absolutely infinitely complex computation that is executed by cosmological natural selection. This computation takes every possible fact into account. Every actual fact has an explanation and has been predetermined to be the case with 100% certainty. Whereas for determinism, everything is determined once a contingent initial state is given, predeterminism starts from the existence of a plenitude.

According to the assumption of Worldly Self-Indication, worlds with a greater abundance, a greater size, and a greater age have a greater probability to be actual, because they materialize a greater fraction of the plenitude. The number of worlds (or types of world duplicates) becomes absolutely infinitely great in the limit to the plenitude. Consequently, also the differences between the total size-ages of worlds in the plenitude become absolutely infinitely great.

If a world x has a total size-age that is infinitely smaller than that of a world y, then the probability of x to be actual becomes zero. This reduces the probability to be actual to zero for most worlds in the plenitude. However, for two cases it can easily be explained that their total size-ages are maximal, such that their probabilities to be actual are also maximal: for the minimum world (a point during one instant of time) and for the plenitude itself. Their total size-ages are equal to that of the plenitude, and their probabilities to be actual are one. They materialize 100% of the plenitude.

According to the proof of predeterminism, there is a partonomically complete sequence of worlds, or a world life, in between these two cases, such that their total size-ages are also maximal, namely equal to that of the plenitude. The probabilities to be actual of all the worlds in this world life are all equal to one. All other world lives that live in parallel with the certainly actual world life, materialize only an infinitesimally small fraction of reality, and are therefore non-actual with certainty. However, they can become certainly actual by merging with the certainly actual life.

## Acknowledgments

I thank Ludger Jansen and two anonymous reviewers for their feedback.

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