Prolegomena in Plato

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Abstract: The article demonstrates unity in Plato's thought to a degree not heretofore realized and suggests analytical links to developments in logic, metaphysics and epistemology millennia later, substantiating Whitehead's famous comment that 'the safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato.'

Keywords: Plato, Aristotle, Kant, *Phaedo*, *Sophist*, *Theaetetus*, *Cratylus*, Forms, predication, naturalistic fallacy, the ethics of belief, relativism, counterfactuals, syllogistic logic.

Overview

This article seeks to create a new philosophical discipline that might be called 'analytical history of ideas', similar in scope and intent to Danto 1968 and Danto 1973 and hinted at in Cusmariu 2022. Key passages in *Sophist*, *Theaetetus* and *Cratylus* are linked with the centerpiece, a famous passage in the *Phaedo*, showing that insights collectively represent a unity in Plato's thought not heretofore realized and foreshadow concepts and theories formulated millennia later in logic, metaphysics and epistemology.[1]

Four Prescient Passages

1. *Phaedo* 100c-e, Grube Translation

The article discusses sentences 1-8, omitting italicized sentences for the reasons stated.

> **Sentence 1**: If there is anything beautiful besides the Beautiful itself, it is beautiful for no other reason than that it shares in that Beautiful.
>
> **Sentence 2**: I say so with everything.
>
> **Sentence 3**: Do you agree to this sort of cause? -- I do.

I no longer understand or recognize those other sophisticated causes.

- This sentence seems to be a sarcastic preamble to Sentence 4.

[1] A follow-up article is planned that will cover prolegomena in ethics, politics and aesthetics as well as other issues in logic, metaphysics and epistemology.

> **Sentence 4**: If someone tells me that a thing is beautiful because it has a bright color or shape or any such thing, I ignore these other reasons – for all these confuse me.
>
> **Sentence 5**: I simply, naively and perhaps foolishly cling to this, that nothing else makes it beautiful other than the presence of, or the sharing in, or however you may describe its relationship to that Beautiful we mentioned.
>
> **Sentence 6**: For I will not insist on the precise nature of the relationship.

Only that all beautiful things are beautiful by the Beautiful.

- Sentence 1 already captured the content of this sentence.

> **Sentence 7**: That, I think, is the safest answer I can give myself or anyone else.
>
> **Sentence 8**: And if I stick to this I think I shall never fall into error.

This is the safe answer for me or anyone else to give, namely, that it is through Beauty that beautiful things are made beautiful.[2]

- Previous sentences already captured the content of this sentence.

2. *Sophist* 259e, Akrill Translation

It is because of the interweaving of Forms with one another that we come to have discourse.[3]

3. *Cratylus* 402a, Reeve Translation

Heraclitus says somewhere that 'everything gives way and nothing stands fast,' and, likening the things that are to the flowing of a river, he says that 'you cannot step into the same river twice.'[4]

4. *Theaetetus* 152a, Cornford Translation

He [Protagoras] puts it in this sort of way, doesn't he, that any given thing 'is to me such as it appears to me, and is to you such as it appears to you.'[5]

***Phaedo* Sentence 1**

[2] G.M.A. Grube (1899-1982) was a major Plato scholar. He also published translations of *Republic*, *Euthyphro*, *Apology*, *Crito*, and *Meno* and authored Grube 1935. I consulted eleven other translations of the *Phaedo* passage to make sure my analysis avoided translation bias: Jowett 2023 (1892), 92; Horan 2023, 44; Emlyn-Jones and Preddy 2017, 457-459; Long 2010, 95; Brann, Kalkavage and Salem 1998, 80; Gallop 1993, 56-7; Larson 1980: 96; Tredennick 1961, 81-82; Bluck 1955, 114-115; Fowler 1914, 345; and Church 1903 (1880), 182-183.

[3] I also consulted translations of this sentence in White 1997; Brann, Kalvage & Salem 1996; Benardete 1984; Cornford 1961; and MacKay 1868.

[4] I also consulted translations of this sentence in Jowett 1961 and Fowler 1926.

[5] I also consulted translations of this sentence in Horan 2021 and Levett 1997.

If there is anything beautiful besides the Beautiful itself, it is beautiful for no other reason than that it shares in that Beautiful. [Grube 1997, 86]

- **Prolegomena**

1. Syllogistic Logic Challenges

An Imaginary Dialogue

SOCRATES: As we work our way through problems, I will be presenting arguments for or against various and sundry solutions. I will, of course, do my best to make sure those arguments are logically correct. I must admit, however, that I don't at the moment have a method, other than the one named after me, for evaluating the logical correctness of arguments. We have with us a new student who hails from up north in Stagira and has expressed a keen interest in this sort of problem.
ARISTOTLE: I'll look into it eventually. There is a lot to learn in the meantime.
SOCRATES: Of course, of course.
PHAEDO: Whatever you figure out, you better make sure it will help us geometers with our proofs. By the way, I understand you're not a geometer, is that right?
ARISTOTLE: I'm not a geometer.
PHAEDO: In that case, perhaps you can start with something simple. What is the logical form of 'Everybody loves somebody sometime'?
SOCRATES: Our colleague has a wicked sense of humor.
PHAEDO: I wasn't joking.
SOCRATES: Patience, Phaedo, patience; let's give our colleague the benefit of the doubt.

An adequate system of logic should be able to confirm the validity of arguments that seem intuitively to be valid, either in the semantic sense of showing that the conclusion cannot be false if the premises are true; or in the syntactic sense of showing that arguments instantiate an axiom or a valid rule of inference.

With that in mind, let us put to a test Kant's well-known comment (1929, 17 B viii) that syllogistic logic (SL) "… is thus to all appearance a closed and completed body of doctrine."[6] The tests I have in mind are much simpler than Phaedo's requirement that SL help geometry with its proofs.

[6] Here is a key passage of the Deduction in *Critique of Pure Reason* (B 131-132): "It must be possible for the 'I think' to accompany all my representations; for otherwise something would be represented in me which could not be thought at all, and that is equivalent to saying that the representation would be impossible, or at least would be nothing to me." Trying to formulate the argument in this passage in SL should have persuaded Kant that SL was far from 'a completed body of doctrine'. I note with some amusement that SL was not one of the 'related systems' Gödel targeted in his 1931 paper "On Formally Undecidable Propositions of Principia Mathematica and Related Systems", from which we should not infer that SL is 'a completed body of doctrine'.

- First Test

Logoi being made possible by the 'interweaving of Forms with one another' according to *Sophist* (more on this later), we might well expect a property true of the Forms, self-participation, to have a counterpart true of *logoi*, self-entailment. Seems obvious, doesn't it? Well, Aristotle missed it.

We know that every proposition in the propositional calculus logically implies itself.[7] So, can SL confirm the validity of an argument from Sentence 1 to itself?

SL lacks the resources to confirm the semantic validity of this argument. Confirming syntactic validity in SL means showing that an argument from Sentence 1 to Sentence 1 is a substitution instance of a valid syllogism,[8] of which there are 15 in SL, consisting of propositions having a subject term S, a predicate term P and a middle term M, each occurring twice in four specified patterns called 'moods', prefixed by the quantifiers 'All', 'Some' or 'No'. Brackets enclosing each term type will facilitate subsequent analysis: <S>, {P} and [M].

> **Sentence 1**: If there is anything <beautiful besides the Beautiful itself>, it is {beautiful for no other reason than that it shares in that Beautiful}.[9]

Therefore,

> **Sentence 1**: If there is anything <beautiful besides the Beautiful itself>, it is {beautiful for no other reason than that it shares in that Beautiful}.

SL cannot show this inference to be syntactically valid as stated. The reason is simple: SL studies the validity of arguments consisting of two premises and a conclusion, for a total of three propositions. So, would adding a third premise solve this problem? The distribution of terms in the resulting argument structure would have to be this:

> <S> {P}
>
> [M] [M]

Therefore,

> <S> {P}

[M] [M] must be the structure of the new premise because <S> and {P} have already occurred twice, so there is only 'room' for the middle term to occur twice in the minor premise. However, no categorical proposition in standard form contains the same term in subject as well as predicate position. Moreover, neither the rule of distribution nor a Venn diagram can confirm the validity of this

[7] Whitehead and Russell 1910, 103, refers to $\vdash p \supset p$ as the 'principle of identity', from which it follows that $p \vdash p$.

[8] Frege's substitution theorem provided for the first time a formally adequate explanation of the reason why an argument stated in words or symbols is valid by virtue of being a substitution instance of a valid rule of inference.

[9] I am deliberately oversimplifying, realizing that Sentence 1 is in conditional, not SL form.

argument structure. Thus, SL fails to confirm the validity of what is perhaps the most elementary of inferences of the propositional (*logoi*) calculus. Had Aristotle seen this, the propositional calculus would have been developed much earlier.[10] Explaining the difference between propositional and predicate logic proved to be a truly profound problem, only solved by Frege in the 19th century. He also did away with SL and proved Kant wrong.

- Second Test

Another valid inference Phaedo would have recognized as such is from Sentence 1

> **Sentence 1**: If there is anything beautiful besides the Beautiful itself, it is beautiful for no other reason than that it shares in that Beautiful.

to Sentence 1a

> **Sentence 1a**: If there is anything beautiful, it is beautiful for no other reason than that it shares in that Beautiful.

Once again we only have a two component argument. SL cannot be applied to confirm the validity of the argument from Sentence 1 to Sentence 1a as written, so we must add a second premise, Sentence 1b below. The only valid syllogism that is applicable to the example under study is **AAA-1**, which has the following structure:

> All [M] are {P}
>
> All <S> are [M]

Therefore,

> All <S> are {P}

Let us fill in this structure with categorical propositions in standard form, identifying recurring term types using the bracket method.

> **Sentence 1**: All [things that are beautiful besides the beautiful itself] are {things that are beautiful only because they participate in that beautiful itself}.
>
> **Sentence 1b**: All <things that are beautiful> are [things that are beautiful besides the beautiful itself].

Therefore,

> **Sentence 1a**: All <things that are beautiful> are {things that are beautiful only because they participate in that beautiful itself}. We have been able to confirm the validity of the inference from Sentence 1 to Sentence 1a, but only by adding a near tautology, Sentence 1b, which says,

[10] The Stoics started propositional logic in the 3rd century BC but original writings were lost and the subject languished for centuries. It was reinvented by Peter Abelard in the 12th century.

Sentence 1b*: All <things that are beautiful> are [things that are beautiful & things that are distinct from the beautiful itself], which implies a tautology,

Sentence 1b**: All <things that are beautiful> are [things that are beautiful]!

Imagine Phaedo and his fellow geometers going through such contortions while trying to capture a proof from Euclid in SL![11]

2. Noumenal Priority

The conditional structure of Sentence 1 is explicit in eleven translations and implicit in one, which brings up a question we need to ask right away: What was Plato getting at by using conditional form in Sentence 1? I will argue it was not a mere *façon de parler*.

Plato held that Forms – abstract, transcendent, entities that exist necessarily – had maximal reality and uniquely so. As he also tells us in Sentence 1, the Forms are their own perfect exemplars. Anything else, to the extent it is F, is F only conditionally, hence the conditional structure of Sentence 1. Conditionally on what? Everything else that is F is, is F conditionally on the Forms. Conditionally on the Forms how? Anything that is F, is only F by participating in F-ness, meaning that without the Forms, nothing is F. Without the Forms, we don't even know what it means to <u>be</u> F! (I will return to this point later.)

Millennia before Kant, Plato's Parable of the Cave told us that the Theory of Forms is the best we can do to 'save the phenomena'.[12]

As Plato might have put it, empirical reality is an approximation of mathematical reality. For example, physical laws give the equality symbol, =, the usual algebraic meaning for computational purposes, not always acknowledging that the approximate equality symbol, ≈, is, strictly speaking, what is empirically appropriate, as this would complicate computations significantly (try it). A good illustration of Plato's insight is the ideal gas law, usually stated as pV = nRT, whose actual physical meaning when applied in computations is pV ≈ nRT. R, the Avogadro/Boltzmann constant, is one of many physical constants that have an exact mathematical value but in practice are approximated. The ideal gas law is an approximation of the behavior of gases under many conditions and has several limitations. For example, the higher the pressure, the more wrong it is. It would

[11] Cusmariu 2016, 282-285, reconstructs Euclid's proof of Proposition III.6 using the powerful tools of modern logic. Doing so in SL, even if possible, would have been absurdly cumbersome.

[12] Compare Rovelli 2011, 81: "Plato is moving in the right direction: it is by means of mathematics that the physical word will be efficiently described"; and also (2011, 82) "… it was Plato who posed the question 'Can we account for the strange movements of the planets in the sky in terms of some simple and orderly motion?' This was the fateful question that would give rise to Greek mathematical astronomy and, eventually, Copernicus, Kepler, Newton, and all of modern science. It was Plato who insisted that astronomy could and must become an exact mathematical science."

have to be revised with each gas to remain even approximately accurate. If you can hear Plato saying, 'I told you so', you are right.

The 'One-Over-Many' model 'saves the phenomena' more efficiently than Kant's Thing-in-Itself because the Forms also solve the problem of universals, which Kant did not.[13] The first *Critique* doesn't even hint at one of the oldest problems in philosophy![14] As to the relation between Forms[15] and empirical exemplars, about which Plato wavers later in our passage, I will construe participation (also) as a kind of substitution in the logical sense, so that Forms also have a logical function. All in due course.

3. Negative Existentials

What might be called 'the Riddle of Nonbeing' has been a poser since Plato, of which the *locus classicus* is perhaps *Sophist* 259A. Let us state the problem in argument form.[16]

(1) If *x* exemplifies property F, then *x* is F.

- This is a near tautology.

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[13] Kant's famous objection to the Ontological Argument is that existence is not a 'real predicate', by which he meant existence is not a 'real property', rather than not a linguistic item of the predicable kind. The larger problem is to explain what a property is, which means solving the problem of universals. Calling existence not a 'real property' simply dodges this problem. See Van Cleve 1999, 188.

[14] Referring to the Forms, Kant writes (1929, 310): "For Plato ideas are archetypes of the things themselves." This (a) is the sort of crude psychologism one might hear in a freshman philosophy class; (b) gets the relationship between Forms and empirical exemplars exactly backwards; (c) is refuted by the modern conception of properties as abstract 'One-in-Many' universals; (d) is refuted by a correct understanding of Plato's role in shaping the mathematical orientation of modern science as explained in Rovelli 2011.

[15] It's a good question whether ontology needs suprasensible objects categorically distinct from and fully independent of the world of appearances, Platonic Forms; as well as Kantian Things-in-Themselves, suprasensible objects that in some sense exist 'behind' and are also independent of 'the veil of appearances.' I don't think so but this is not the place to argue the matter. Kant's misunderstandings of Plato evidently prevented him from taking a hard look at this issue. Kant scholars such as Van Cleve 1999, Hartnack 1967, Strawson 1966, Bennett 1966, Bird 1961, Weldon 1958, and Körner 1955 do not address the issue. Walsh 1947 (36, 56, 101-103) notes key aspects of the Theory of Forms but not as an ontological alternative to Things-in-Themselves. None of the articles in Wolff 1967 address the issue. Of the articles in Guyer 1997, only O'Neill talks about Plato but all she does is tell us (285) that Kant rejected "the entire Platonic account of the metaphysical basis of unity", as well as "all thought that his Ideas of Reason correspond to any real archetypes, and adopts a position that is irreconcilable with any form of the Platonic vision of Ideas as patterns for knowledge and mathematics." O'Neill repeats Kant's canard about Forms being 'archetypes' and adds a new one, 'patterns', leaving both unexplained.

[16] See Cartwright 1960; Owen 1970; Wiggins 1970; Plantings 1974; Gale 1976; and Cusmariu 1978C.

(2) If *x* exemplifies property F, then *x* exists.

- This is another near tautology.

(3) If *x* exemplifies *nonexistence*, then *x* exists.

- This follows from 2 by substitution.

(4) If *x* exemplifies *nonexistence*, then *x* does not exist.

- This follows from the meaning of the property *nonexistence*.

(5) If *x* exemplifies *nonexistence*, then *x* exists and does not exist.

- This follows from (3) and (4).

(6) *Nonexistence* is not a property.

- This allegedly follows from (5).

However, what follows from (5) is the tautology that everything exists,

(7) *Nonexistence* is unexemplified.

To derive (6), we would need

(8) There are only properties that are exemplified,

which Aristotle may have held (see Cresswell 1975 and my critique of this article in Cusmariu 1979A.)

What follows from the fact that *nonexistence* is not a property of anything is that negative existentials such as

(10) Santa Claus does not exist

and

(11) There are no roundsquares

cannot be analyzed via

(12) Santa Claus exemplifies *nonexistence*

and

(13) Roundsquares exemplify *nonexistence*.

This is not a problem for the Theory of Forms, which can analyze (10) and (11) as (12) and (13):

(12) The property *being Santa Claus* is unexemplified.

(13) The property *being a roundsquare* is unexemplified.

Unexemplified (and unexemplifiable) properties are included in the Theory of Forms. This paper can only indicate some of the reasons why such a rich ontology is necessary.

4. Russell's Paradox

An Imaginary Dialogue

SOCRATES: So, there is a Form, Beauty, in which all and only beautiful things participate.
PHAEDO: There is.
SOCRATES: Is there also a Form in which all and only things that are not beautiful participate?
PHAEDO: No. All and only things that are not beautiful simply do not participate in Beauty.
SOCRATES: That is not true.
PHAEDO: Why not?
SOCRATES: If something does not participate in Beauty, it might be for reasons having nothing whatever to do with whether it is beautiful.
PHAEDO: I see. You're right.
SOCRATES: So, there is a Form in which things participate because and only because they are not beautiful. We can call it Unbeauty, I suppose.
PHAEDO: That's an odd sort of Form.
SOCRATES: I suppose. Now, every Form has a complement. Things participate in one or the other but not both. Do you agree?
PHAEDO: I do. I have a feeling you're leading up to something. I hope so.
SOCRATES: Patience, Phaedo, patience. I said earlier that Beauty itself is also beautiful, meaning that our Forms are their own perfect exemplars.
PHAEDO: Yes, that's true.
SOCRATES: Does that mean that there is a Form in which all and only Forms participate by virtue of participating in themselves? Call it Self-Participation.
PHAEDO: That's also an odd sort of Form, but I see now reason to think there is no such Form.
SOCRATES: We agreed that every Form has a complement, from which it follows that there is also such a Form as Non-Self-Predicability, right?
PHAEDO: You're inventing all kinds of odd Forms today. Well, I could see no way to stop you from saying there is such a Form as Self-Predicability. I see no way to stop you from saying there is such a Form as Non-Self-Predicability.
SOCRATES: I think you'll change your mind in a minute.
PHAEDO: Why is that?
SOCRATES: If there is such a Form as Non-Self-Predicability, then, it is either self-predicable or it is not. We agreed that was true in general: Something is either red or not-red; round or not-round; and so on.
PHAEDO: Yes.
SOCRATES: It is easy to see that Non-Self-Predicability participates in itself if and only if Non-Self-Predicability does not participate in itself, which is impossible. Therefore, there is no such Form as Non-Self-Predicability, even though it seemed that there might be.

PHAEDO: You are absolutely right. Now what?
SOCRATES: I hope someone will figure out this problem some day.[17]

5. Higher-Order Logic

The One-Over-Many model means that there is also a Form, a One, Over the Many Forms that participate in themselves, namely, Self-Participation. This is but a short step from what we now call properties of properties, which belong to higher-order logic. This consequence of the One-Over-Many model – that SL failed to explore – lay dormant literally for millennia and was only recognized at the end of the 19th century by Frege. However, neither he nor his followers seem to have realized that they were being original only in matters of technical detail. Plato got there first.

Now, consider Forms such as Being a Form, Being Abstract, Being Transcendental and Being a Necessary Existent, which are properties of properties. In formal logic, they are considered third-order properties and would be quantified over in third-order logic. Second-order properties quantified over in second-order logic are properties of mathematical objects (Shapiro 2000).

So, are there are any properties at all that can legitimately be said to be self-exemplified? The answer is, yes. Thus, Being a Property, Being Abstract, Being Transcendental, Being a Necessary Existent, Existing Independently of Particulars and Existing Unexemplified are all evidently self-exemplified. However, we can only write

SE1. F exemplifies F if only if F is exemplified by all properties.

rather than

SE2. F exemplifies F if only if F is exemplified by all <u>and only</u> properties

because numbers and sets are also abstract and exist necessarily.[18] SE1 would 'rescue' Plato's self-participation claim by answering familiar objections to the self-predicability of Forms, though evidently it is not the concept of self-predication Plato intended.[19] The Forms he usually mentions as self-exemplifying, Beauty, Justice and Goodness, are not so according to SE1. How replacing standard self-predication with SE1 would affect Plato's metaphysics generally is beyond the scope of this paper. I bring it up to suggest a consequence of the Theory of Forms

[17] This is the property (propositional function) version of Russell's Paradox (Russell 1902), formulated as a refutation of Frege's Law V, which assumed that any property determined a set. Russell's 1902 letter to Frege stated both versions, though only the set version is formulated symbolically. Cusmariu 1979B discusses the set version.
[18] Unless, of course, numbers and sets reduce to properties.
[19] Malcolm 1991 is a book-length study of self-predication in Plato.

whose significance as an intuitive starting point of higher-order logic was realized only millennia later.[20]

6. Belief *De Dicto* and Belief *De Re*

One criticism that has been raised (including by Aristotle) is that the Theory of Forms is unnecessarily powerful; so powerful in fact that it is its own worst enemy, as the famous 'Third Man' Argument in *Parmenides* allegedly demonstrated (see Cusmariu 1985). However, the criticism proved shortsighted at several junctures in the history of philosophy. It happened again more recently when a property of the form *being believed to have a property* proved useful.

No one suspected that such an unusual property was a consequence of the Theory of Forms, certainly not in the version stated in Sentence 1 of *Phaedo*. We are very far from the simple structure of Sentence 1 to suppose that there could be a Form participated in by all and only those things that are believed to have a property. That, however, is what follows from the Theory of Forms once we grant that there is a Form of F-ness in which all and only things participate by virtue of being F – assuming we can solve the problem of Non-Self-Predicability.

Quine 1956 introduced an important distinction between 'notional' and 'relational' belief. Notional belief is propositional and referentially opaque, *de dicto.* Relational belief is non-propositional and referentially transparent, *de re*, which turned out to make use of a property of the form *being believed to have a property.* Quine did not explain relational belief that way because he thinks (186) properties (intensions) are 'creatures of darkness' that need to be 'exorcised'. I propose to ignore what are essentially *ad hominem* comments.

Thus, in the notional sense, Smith can believe the proposition that the Morning Star is the planet Venus but reject the proposition that the Evening Star is the planet Venus, not realizing that they are identical. Accordingly, Smith would reject the inference of 3 from 1 and 2.

1. Smith believes that the Morning Star is the planet Venus.

2. The Morning Star is identical with the Evening Star.

3. Smith believes that the Evening Star is the planet Venus.

However, in the relational sense, Smith believes <u>of</u> the Morning Star that it is the planet Venus, in which case Smith 'has his candidate' (as Quine put it, 185), so that the Morning Star has the property *being believed by Smith to be the planet Venus*. Because the Morning Star is identical with the Evening Star, every property of one is a property of the other. Accordingly, it follows that the Evening Star has the property *being believed by Smith to be the planet Venus* and the inference from 4 and 2 to 5 goes through.

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[20] Aristotle spent some twenty years at Plato's Academy and eventually produced his own solution to the problem of universals. Cusmariu 1979A takes a hard critical look at (and demolishes) a modern reconstruction of Aristotle's theory of universals in Cresswell 1975.

4. The Morning Star is believed by Smith to be the planet Venus.

2. The Morning Star is identical with the Evening Star.

5. The Evening Star is believed by Smith to be the planet Venus.

By including a property of the form *being believed to have a property*, the Theory of Forms is able to incorporate a useful distinction in the philosophy of mind. It's a good question whether other solutions to the problem of universals can do that.

To forestall potential misunderstanding, we should add that whether or not Smith or the planet Venus exist has no bearing on whether the property *being believed by Smith to be the planet Venus* exists; only on whether this property is exemplified. The Theory of Forms distinguishes sharply between the existence of Forms and their exemplification, including their exemplifiability. Unexemplified as well as unexemplifiable Forms are admitted under the most powerful version of the Theory of Forms – though it is a matter of scholarly debate whether Plato held such a view.[21]

Phaedo Sentence 2

And that is what I say about them all. [Long 2010, 95][22]

- **Prolegomena**

1. A Syllogistic Logic (SL) Challenge

As the expressions 'everything', 'all the others', 'all things', 'all of them', 'every kind of thing', and 'all phenomena' make clear, Sentence 2 is a generalization. What is less clear is whether Plato intended to assert a logical relationship between Sentence 2 and Sentence 1; and if so, in which direction.

[21] A reduction of belief *de re* to belief *de dicto* would need to preserve the inference from 4 and 2 to 5 and overcome Quine's objections to quantifying into opaque contexts. I attempted such a reduction in Cusmariu 1977. Alternatively, the constituent properties of a proposition could function as the objects of a belief *de re* in a reduction of belief *de dicto* to belief *de re*. Thus, "Smith believes with respect to *Socratic wisdom* that it is exemplified' could reduce 'Smith believes that Socrates is wise." If Smith doesn't know that Socrates is the philosopher who drank hemlock, he would reject the proposition that the philosopher who drank hemlock was wise, which rejection would have to be preserved by a reduction of "Smith believes that the philosopher who drank hemlock is wise" in terms of belief with respect to the properties that are the constituents of this proposition. The ontological resources of the Theory of Forms are powerful enough to permit a reduction of belief *de dicto* to belief *de re* with respect to properties of any logical complexity. As to how abstract objects such as properties could be objects of belief or cognition generally, Russell (1912, Ch. X) saw no difficulty in the matter, nor did Church (1951, 104); though it would have to be explained how exactly a property could be a person's 'candidate' in Quine's sense.

[22] Translations in Tredennick 1961 and Tredennick & Tarrant 1954 omit this sentence.

Assuming a logical relationship was intended, there are two possibilities:

1. The inference is particular-to-general: From Sentence 1 to Sentence 2.

2. The inference is general-to-particular: From Sentence 2 to Sentence 1.

Both inferences are intuitively valid. There is only need to study one of them, however. Let us determine whether SL can confirm the intuitive validity of the argument from Sentence 2 to Sentence 1 – modern logic can do both.

Here are the two components of such an argument:

I. Object X is F because and only because X participates in F-ness.

Therefore,

II. Object X is beautiful because and only because X participates in Beauty.

First, we state I and II as **A** categorical propositions in standard form, replacing 'because and only because' with the copula. This yields the following argument:

Ia. All things that are F are things that participate in F-ness.

Therefore,

IIa. All things that are beautiful are things that participate in Beauty.

For reasons already indicated, SL cannot confirm that the inference from Ia to IIa is valid as written. What we must do, again, is add a second premise to complete the **AAA-1** structure. The new premise must conform to the constraints of **AAA-1**, which requires that subject and predicate terms of the conclusion occur one more time in the premises, with the remaining positions being occupied by the middle term. This means that subject and predicate terms are to be based on the terms of the conclusion, IIa. This leads to the following structure:

All M are P: All [M things] are {things that participate in beauty}.

All S are M: All <things that are beautiful> are [M things].

Therefore,

All S are P (IIa): All <things that are beautiful> are {things that participate in beauty}.

From Ia and IIa it follows that the middle term can be either [things that are F] or [things that participate in F-ness], resulting in two syllogisms with the same conclusion:

- Syllogism 1

(i) All [things that are F] are {things that participate in beauty}.

(ii) All <things that are beautiful> are [things that are F].

Therefore,

(iii) (IIa) All <things that are beautiful> are {things that participate in beauty}.

- Syllogism 2

(iv) All [things that participate in F-ness] are {things that participate in beauty}.

(v) All <things that are beautiful> are [things that participate in F-ness].

Therefore,

(vi) (IIa) All <things that are beautiful> are {things that participate in beauty].

Both syllogisms are valid but neither is sound.

- Premises (i) and (iv) are false or lack truth value.
- Premises (ii) and (v) lack truth value as written.
- The predicate F can assign a truth value to premise (ii) if F is defined as 'beautiful.' But then, (ii) becomes a tautology. Any other value would turn (ii) into a falsehood.
- The same thing happens in premise (v) if 'F-ness' is defined as 'Beauty.' So, (ii) and (v) either lack truth value, are tautologies, or are false.

In any case, this is all academic. Neither syllogism is able to capture the inference from Ia to IIa for the simple reason that Ia is nowhere in sight![23]

We have here another example of elementary argumentation whose validity SL cannot confirm. Confirming the validity of the inference from Sentence 2 to Sentence 1 is especially critical because it goes to the heart of the Theory of Forms.[24] As it turned out, Socrates' confidence in Aristotle was misplaced.

Phaedo Sentence 3

Do you accept this kind of causality? Yes, I do. [Tredennick 1961, 81.]

- **Prolegomena**

1. Causation

In a paradigm case, causation holds between events (Kim 1993). Events imply change: something exemplifies a property at time t it did not exemplify prior to t; or, no longer exemplifies a property at t it exemplified prior to t. Here is a paradigm case of causation:

E1. A steel ball struck a window pane at t.

E2. The window pane shattered shortly after t.

[23] SL cannot confirm the validity of the inference from Sentence 1 to Sentence 2 for the same reasons.

[24] The reader who carries out this argument with another valid syllogism, **AII-1**, will reach the same conclusion.

We describe the situation in a paradigm case by saying that E1 caused E2. In E1, the window pane exemplifies at t the (complex) property of being struck by a steel ball, a property (let us suppose) it did not exemplify prior to t. In E2, the window pane exemplifies shortly after t the (complex) property of being shattered, a property it also did not exemplify prior to t.[25]

So, Platonic Forms can be causes in a paradigm sense of causation provided we can construe Platonic Forms as (a) properties that (b) can be constituents of events in some sense of 'constituent'. It is reasonable to give Plato credit for being farsighted enough to see that Forms could be causes in paradigm cases of causation. The technical details involved in spelling out (a) and (b) are for another time.[26]

2. Recurrence

Plato postulated Forms in part as a response to a doctrine of an illustrious predecessor, Heraclitus, namely, his 'flux' thesis.[27] Here are three translations of this thesis that Plato states in *Cratylus* at 402a:

- Heraclitus says somewhere that 'everything gives way and nothing stands fast,' and, likening the things that are to the flowing of a river, he says that 'you cannot step into the same river twice.' [Reeve 1997, 120]
- Heraclitus is supposed to say that all things are in motion and nothing at rest; he compares them to the stream of a river, and says that you cannot go into the same water twice. [Jowett 1961, 438]
- Heraclitus says, you know, that all things move and nothing remains still, and he likens the universe to the current of a river, saying that you cannot step twice into the same stream. [Fowler 1926, 67]

Because a river is a collection of stages defined by spatio-temporal properties, there is a sense in which Heraclitus' famous claim is trivially false: It is indeed possible to step into the same river twice; just not the same river stage. We can take it for granted, however, that Plato understood as much and was not motivated to postulate eternal, unchanging Forms because he failed to see the obvious.

Heraclitus's doctrine of world impermanence raises questions about identity through time that Plato can be understood as seeking to answer, using the resources of his Theory of Forms. It may appear as if nothing is the same from moment to moment but this is not literally true. Countless properties and relations are exemplified and co-exemplified again and again, and continuously so, and are

[25] I will set aside (a) what events are; (b) identity conditions for events; (c) how properties can be constituents of events; (d) whether there are logically complex properties; and (e) the generic-specific distinction for events.

[26] See below for an imaginary dialogue that links Forms to counterfactuals.

[27] The rock band The Young Rascals echoed an epistemic version of Heraclitus in an oft-quoted 1967 lyric: "How can I be sure in a world that's constantly changing?" How indeed.

the reason (*aitia*) why the world 'hangs together.' Properties play a key role in the identification and reidentification of ordinary objects such as tables and chairs from hour to hour and day to day. Moreover, as constituents of events, properties can explain what it means for events to be the same, or not; and how it can be that we can step into the same river twice but not the same river stage: The same properties are exemplified in one case but not the other because different spatio-temporal river stages are defined by different spatio-temporal properties. Recurrence, after all, is one of the three components of the problem universals; the other two are predication and classification (see Cusmariu 1979A).

Phaedo Sentence 4

If someone tells me that a thing is beautiful because it has a bright color or shape or any such thing, I ignore these other reasons – for all these confuse me. [Grube 1997, 86]

Having stated in Sentence 1 his view that things are beautiful because they participate in Beauty and for no other reason, Plato would, of course, reject any alternative reason as a matter of elementary logic.[28]

However, that is not what Sentence 4 says.

Sentence 4 does not say or imply that:

(a) 'these other reasons' are wrong;

(b) 'these other reasons' cannot be true if his Theory of Forms is true; or

(c) 'these other reasons' are unnecessary if the Theory of Forms is sufficient.

Rather, Plato's Sentence 4 says that 'these other reasons confuse me'. When a great philosopher, who could just as easily have asserted (a), (b) or (c), asserts instead that he finds a certain view confusing, we need to take him at his word and find out why. What confusion or confusions might Plato have been getting at?

A second issue concerns the specific examples of 'those other reasons' cited, namely, 'having a bright color or shape.' Is there a special significance to these examples? If so, what is that? I will start with this issue.

- **Prolegomena**

1. Primary and Secondary Qualities

Plato sometimes makes philosophically important points in an off-handed, almost casual way, inviting the audience to puzzle them out rather than spoon-feed a

[28] The argument from the proposition that the Theory of Forms is sufficient to account for X to the proposition that other theories are unnecessary to account for X is another intuitively valid inference that SL cannot confirm for reasons that are analogous to those already presented, as the reader can easily verify.

solution. As Wittgenstein put it (1953, x), "I should not like my writing to spare other people the trouble of thinking."

So, what philosophically important point is being made by listing color and shape properties together as reasons why something is beautiful, which Plato rejects as confusing? After all, he could have listed only shape properties, or only color properties, or neither.

Plato would have associated shape properties with geometry, whose propositions he considered to be true irrespective of whether they have been proved, thought about and so on. Accordingly, he considered shape properties to hold true irrespective of a perceiver, as intrinsic to whatever object has them, to be give an *aitia* the way all of mathematics can.

On the other hand, there are properties that depend on the ability of the perceiver to be affected by them and as such are relational, extrinsic properties, requiring a different *aitia*.

I am suggesting that Sentence 4 anticipates a distinction drawn by Locke and others between primary and secondary qualities. Shape properties are primary, while color properties are secondary. Plato tells readers he finds the exemplification of such properties confusing as an *aitia* of what it is to be beautiful to warn that there is a difference in kind between shape and color properties and to invite inquiry into the difference.

Well-known articles and books on the *Phaedo* do not address the issue.[29]

- Burnet (1911, 111) limits his comments on Sentence 4 to color properties, evidently seeing no reason to remark on the fact that Sentence 4 mentions shape properties as well.
- Vlastos 1970 contains an extensive discussion of reasons and causes in the *Phaedo* but does not see that Sentence 4 implies a distinction between primary and secondary qualities.
- Irwin (1999, 166) muddies the waters by lumping shapes and colors together as 'sensible' properties.
- M-K Lee's cites evidence (Nolan 2011, 28-31) that Plato distinguished primary from secondary qualities in a late-period dialogue, the *Timaeus* at 61c-d. She comments (28) that Plato would not have drawn this distinction in a middle-period dialogue such as the *Phaedo* because during that period "Plato notoriously holds that perception is systematically misleading about a systematically misleading part of reality".
- Emlyn-Jones and Preddy (2017, 282) breeze past Sentence 4 entirely, limiting their comments to restating the Theory of Forms.

[29]Book-length commentaries include Ritchie 1902, 95; Shorey 1933, 179; Crombie 1963, Ch. 2, Section II; Field 1969, 29, 30; White 1976, Ch. III; Grube 1980, 19; Hare 1982, 45; Dorter 1982, 128; Bostock 1986, Ch. VII; Stern 1993 and Ebrey 2023, 230. There is no discussion of the issues of interest here in Ahrensdorf 1995.

2. Predication v. Predicates

Another distinction that should not be confused according to Sentence 4 is between

- An (a)-*aitia*, which explains what is for a term F to be predicated of an object X; that is, what it means of X to be F;

and

- A (b)-*aitia*, which unpacks the semantic content of F; that is, what it means for X to be F as opposed to G or H or ...

The Theory of Forms is an (a)-*aitia*, while definitions carry out (b)-*aitiai*. Of course, unpacking the semantic content of a term must necessarily involve predication with respect to each and every component of a (b)-*aitia*, so that Forms are exemplified in any case.

To put these points another way, consider an argument that has the logical form of an **AAA-1** syllogism:

1. If X is beautiful, then X is FGH associated with being beautiful.

2. If X participates in Beauty, then X is beautiful.

Therefore,

3. If X participates in Beauty, then X is FGH associated with being beautiful.

We must be careful not to confuse the (a) and (b)-*aitiai*. Thus, premise 1 is part of (b)-*aitiai*, while premise 2 is part of (a)-*aitiai*. We can avoid confusion by restating the argument using copulas linked to the appropriate *aitia*.

1. If X participates in Beauty, then X is-(a) beautiful.

- Plato would accept premise 1 because it is entailed by an (a)-*aitia*.

2. If X is-(a) beautiful, then X is-(b) FGH associated with being beautiful.

- Plato already rejected premise 2 as one of 'these other reasons' that he finds 'confusing'.
- Replacing the antecedent of premise 2 with 'If X is-(b) beautiful' results in a true premise but the argument is no longer valid because the consequent of premise 1 reads 'X is-(a) beautiful'.
- Replacing the consequent of premise 1 with 'X is-(b) beautiful' results in a false premise.

Therefore,

3. If X participates in Beauty, then X is-(b) FGH associated with being beautiful.

- Plato would have rejected 3 because an (a)-*aitia* does not entail a (b)-*aitia*.

3. Is the Copula Eliminable?

Perhaps a key component of the problem of universals, predication, is not a genuine problem because the copula is eliminable by paraphrase, or so it might be thought.

One way to try to eliminate the copula by paraphrase is to turn a copula-adjective phrase such as 'is beautiful' into the verb 'beautifies', so that "Bethany is beautiful" becomes "Bethany beautifies". Well and good except that we are now entitled to ask "what does she beautify?" After all, there is usually a value of G such that X FGs, as in "Bethany beautifies her room". So, turning a copula-adjective phrase into a verb merely gets rid of monadic predicates in favor relational ones, as Russell pointed out (1912, 97) in his critique of Berkeley and Hume. The paraphrase proponent can reply at this point that his use of 'beautifies' is non-standard and as such does not require an object. Without further elucidation, however, this reply is merely a dodge.

We can bring out the relationality of verbs further by noting that adverbs can modify verbs, as in "Smith ran quickly", making comparative sentences possible such as "Smith ran more quickly than Jones." Thus, "Bethany beautifies (a room?) more efficiently than Annette" also only eliminates monadic predicates and corresponding properties, leaving relational predicates and corresponding relations unaffected, and we are back to square one. The progress represented by paraphrasing sentences of the form "X is (predicatively) F" into sentences of the form 'X Fs' is an illusion.

4. Realism v. Nominalism

Math and science books routinely write 'F(x),' spell out the details of the function being defined, i.e., (b)-*aitiai*, and then get on with the business of computing the value of the function as if the predication implicit in this notation, an (a)-*aitia*, need not be addressed and along with it the problem of ontological commitment to abstract entities. There is a sense in which this is true and a sense in which it isn't.

- Computing the value of a function by means of a calculator or similar device means entering numerical values, pressing a key and letting the calculator do the rest. Obviously, there is no way to enter into a calculator the details of an (a)-*aitia* or information about kinds of abstract entities! Even if there were, how would a calculator use such information to produce an answer?

- Claude Shannon, Alan Turing and John von Neumann – founders of computer science – evidently did not think they needed to solve the problem of universals and the problem of ontological commitment to get the job done. Solving these problems would not have helped them in the least to work out the many details of a digital computer.

- In the Preface to the first edition of *Critique of Pure Reason* (1929 [1781], Avii) Kant famously stated: "Human reason has this peculiar fate that in one species of its knowledge it is burdened by questions which, as prescribed by the very nature of reason itself, it is not able to ignore, but which, as

> transcending all its powers, it is also not able to answer." Evidently Kant was not 'burdened' by questions about the problem of universals and the problem of ontological commitment and was 'able to ignore' them.

What about this?

- Points 1 and 2: No philosopher would argue that practical applications of concept C are possible only if philosophical problems raised by C are addressed; so that 1 and 2 are not objections to the importance of those problems. However, Alonzo Church was a founder of computer science and took commitment to abstract entities seriously. See Church 1951.
- Point 3: Kant's misunderstanding of the Theory of Forms may well have prevented him from looking into the reasons that motivated Plato's solution to the problem of universals and the related problem of ontological commitment.[30] Interestingly, none of the references to Aristotle in the first *Critique* mention his solution to the problem of universals.

5. The Naturalistic Fallacy

We know that Plato rejects a definition of goodness in terms of properties of actions or their consequences; so that the point made in Sentence 4 about Beauty and not being analyzable in terms of shapes and colors should not come as a surprise. The fact that the context is aesthetics rather than ethics does not matter because the issue at bottom is whether normative concepts can be analyzed in non-normative terms. So, when Plato evinces confusion, he is warning that normative concepts are *sui generis* and cannot be analyzed 'without remainder' into non-normative concepts. This is a modern lesson found in G.E. Moore's *Principia Ethica*.

As we would now put it, definitions are to provide logically necessary and sufficient conditions and not merely true material biconditionals; meaning that counterexamples, which Socrates is more than happy to provide time and again, are sufficient to refute definitions. Thus, a definition of Beauty in terms color and/or shape properties, no matter how well chosen, would at most be contingently true. Citing color and/or shape properties may help decide in a specific case whether an object is beautiful, but that does not entail an analytic connection between a normative concept and properties used to apply it to cases.

6. Relativism

The famous Protagorean thesis that 'man is the measure of all things' (MM) is stated in the *Theatetus* at 152a and is explained shortly thereafter. Here are three translations:

[30] If I had to guess, I'd say Kant was some sort of conceptualist; but what kind would be pure speculation. The references to Hume in the first *Critique* do not mention Hume's opposition to abstract entities.

- Does this not somehow mean that since you and I are men, such as any particulars appear to me, so they are for me, and such, in turn, as they appear to you, so they are for you? [Horan 2021, 11]
- Then you know that he [Protagoras] puts it something like this, that as each thing appears to me, so it is for me, and as it appears to you, so it is for you. [Levett 1997, 169]
- He [Protagoras] puts it in this sort of way, doesn't he, that any given thing 'is to me such as it appears to me, and is to you such as it appears to you.' [Cornford 1961, 856]

The ultimate refutation of MM is the Parable of the Cave. Dwellers are indeed such that 'whatever appears to them' as they look at the shadows dancing on the cave wall, 'is so to them'. The Forms are their only chance to break out of 'the circle of their own ideas' (Leibniz), which can only happen outside the Cave. The truth may be hard to believe even when it is staring them in the face, especially if an epistemic reorientation is necessary – which it is.

To connect MM to the problem at hand, here are our two competing *aitiai* of what it is to be beautiful.

(A) If X is beautiful, then X is beautiful if and only if X participates in Beauty.

(B) If X is beautiful, then X is beautiful if and only if X is FGH, 'these other reasons'.

There is no way for MM to sow confusion in *aitia* (A). There can at most be disagreement about whether or not something is beautiful and, in consequence, whether or not something participates in Beauty. There is nothing subjective about whether one and the same Form of Beauty is participated in. There is no 'Beauty for me' or 'Beauty for you'. There is only Beauty. The Form of Beauty, being a 'One-Over-Many', is the same Form in which it appears to both of us that X participates by being beautiful.

Applied to *aitia* (B), 'these other reasons', MM can easily sow confusion. On the right half of (B) we have a list of perceptible properties about shape and color and the like, regarding which confusing what it is for something to <u>be</u> FGH with what it is for something to <u>appear to be</u> FGH is not only possible but routine. Perceptible properties FGH thought to be associated with beauty can vary from person to person, from culture to culture – indeed, from era to era.

Phaedo Sentences 5 and 6

I simply, naively and perhaps foolishly cling to this, that nothing else makes it beautiful other than the presence of, or the sharing in, or however you may describe its relationship to that Beautiful we mentioned. For I will not insist on the precise nature of the relationship. [Grube 1997, 86]

- **Prolegomena**

1. Participation is Indefinable

In a well-known essay, Gregory Vlastos writes in reference to Sentences 5 and 6 (1970, 142):

> Here is something Plato has not yet cleared up to his satisfaction, though he doubtless expects he will, remaining quite certain for the present that some such relation exists and that, were it not for this, the fact that things have characters, would be unintelligible. [Continues in footnote 31] Though the expectation was never adequately fulfilled, Plato retained the confidence that somehow or other things must 'participate' in the Forms. In the *Parmenides*, at the end of the second regress argument, Parmenides does not conclude that the notion of participation has been invalidated, but only that 'we must look for some other way [i.e., other than similitude] by which they participate' (133A5-6).

Rather than constituting evidence that 'Plato has not yet cleared up to his satisfaction' the relationship between Forms and their instances, the occurrence of several terms in our *Phaedo* passage that express this relationship seems to me anticipatory of a much later development due to Wittgenstein 1953: some terms may form collectively a kind of 'equivalence class,' bearing to each other what Wittgenstein called a 'family resemblance,' each term and its application offering unique intuitive perspectives on an important point about definitions.

Thus, it would clearly be circular to try to define any member of the equivalence class of terms Plato lists in our passage by means of another member of the class (the two starred items at the end are modern terminology.)

- partaking of Beauty
- sharing in Beauty
- participating in Beauty
- presence of Beauty
- association with Beauty
- partnership with Beauty
- communion with Beauty
- communication with Beauty
- exemplifying* Beauty
- instantiating* Beauty

The circularity point also applies to attempted definitions of the relation between Forms and their instances. Thus, suppose we tried to define participation by specifying properties alleged to be necessary and sufficient:

> R is the participation relation =df R is F, G, H …

Predication evidently occurs in the definiens and as such would require the sort of expansion entailed by the Theory of Forms:

> R is the participation relation if and only if R participates in Forms F-ness, G-ness, H-ness…

It is, of course, impossible to define everything, meaning that some terms or concepts must be assumed as primitive. Why not participation, or its modern equivalent, exemplification? That said, the fact that circularity would plague any definition of participation does not mean we cannot say anything interesting about its logical properties *qua* relation. For example:

- Participation is not transitive. Thus, red roses participate in the property of being red and the property of being red participates in the property of being a property; but red roses do not participate in the property of being a property.
- Participation is not symmetric. Thus, red roses participate in the property of being red but the property of being red does not participate in red roses.
- Participation is not reflexive. Thus, participation relates forms to themselves according to Plato, but this is not true in general. Red roses do not participate in red roses.

Though Aristotle lists relations among his categories, the hard work required to spell out the logic of relations had to wait literally millennia (see McBride 2020.)

2. Participation as Substitution

In the *Phaedo* and elsewhere, Plato asserts the familiar conception of Forms as Ones in which Many participate as part of his solution to the problem of universals. However, in the *Sophist* at 259e, Plato can be interpreted as suggesting an altogether different conception of Forms that sheds new light on the concept of participation that Plato found difficult to define in the *Phaedo*. Here are five translations of the now-famous *Sophist* passage:[31]

- The weaving together of forms is what makes speech possible for us. [White 1997, 263]
- For speech has arisen for us through the interweaving of the forms. [Brann, Kalkavage & Salem 1996, 71]
- For it's on account of the weaving together of the species with one another that the speech has come to be for us. [Benardete 1984, II.57]
- Any discourse we can have owes its existence to the weaving together of forms. [Cornford 1961, 1007]

[31] Immediately prior to the 'interweaving' metaphor, Plato writes [Akrill 1970, 201]: "the isolation of everything from everything else is the total annihilation of all statements." This can be seen as a precursor of Frege's influential Context Principle [1884/1980, x]: "Never ask for the meaning of a word in isolation, but only in the context of a proposition."

- It is because of the interweaving of Forms with one another that we come to have discourse. [Akrill 1970, 201]
- It is through the mutual interlacement of ideas or forms that discourse becomes possible. [MacKay 1868, 165]

Here Forms are not invoked to explain the familiar components of the problem of universals, such as predication, recurrence and classification. Rather, a strong claim is made to the effect that *logos* – variously translated as 'speech' or 'discourse', though 'sentence' or 'statement' would also be appropriate – is impossible without 'the interweaving of Forms with one another.' This metaphor turns out to have been extraordinarily farsighted. Plato can be interpreted as suggesting that 'the interweaving of Forms with one another' is what make possible meaning, logical form and validity itself later spelled out in first-order logic.

3. Sentential Calculus Application

To illustrate, on left is the familiar rule of the sentential calculus, Hypothetical Syllogism (HS).[32] On right, is the result of replacing colors uniformly with *logoi*.[33]

(<…>) → ([…])	If <X is a rose> then [X is a flower]
([…]) → ({…})	If [X is a flower] then {X is fragrant}
--------	--------
(<...>) → ({…})	If <X is a rose> then {X is fragrant}

The 'interweaving forms' language can be interpreted as suggesting that Forms in *Sophist* are predicables in a different sense than in *Phaedo*.

> This is the material conditional Form (…) → (…), instantiated in all three schematic sentence-forms on the left.
>
> The three schematic sentences are substitution instances of the material conditional Form in the familiar sense.
>
> Simple substitution instances of the material conditional Form include (<…>) → (<…>) and (<…>) → ({…}).

The material conditional Form (…) → (…) is a Form in Plato's sense as a One-in-Many because it can have instances, even if in a technical sense because *logoi* are what belong inside the brackets rather than non-linguistic objects.[34]

- There are Forms corresponding other logical connectives as well: (…) & (…), (…) v (…), and ~(…).

[32] My book book *Logic for Kids* (Cusmariu 2023) explains the concept of logical form using colors, making it easier for children to follow.
[33] Brackets here and below enclose sentences rather than the three types of terms of SL.
[34] I realize there is a token-type issue here, discussed below.

- Accordingly, (<…>) → (<…>) and (<…>) → ({…}) can be said to participate in the Material Conditionality Form (…) → (…) by being proper substitution instances of it.
- The substitution instances of Material Conditionality Form are other Forms.
- In this extended sense, (<…>) → (<…>) and (<…>) → ({…}) are also Forms, though their instances are *logoi*.
- Instances of the Material Conditionality Form (…) → (…) such as (<…>) → (<…>) and (<…>) → ({…}) can be understood as properties of a paradigmatic property.

Writing HS as a sequence of forms yields

- ((<…>) → ({…}) & ({…}) → ([…])) ► (<…>) → ([…]), where ► means 'logically implies.'
- A truth-table will show that the result of replacing this symbol with a material conditional symbol, →, is a tautology, confirming that HS is a valid rule of inference.
- The sequence ((<…>) → ({…}) & ({…}) → ([…])) ► (<…>) → ([…]) shows the sense in which the three material conditional forms of HS 'interweave with one another' truth-functionally.

'Interweave' in the HS sequence has two meanings we should distinguish carefully.

1. The two material conditional forms (<…>) → ({…}) & ({…}) → ([…]) interweave with one another as substitution-instances of the Conjunctivity Form (…) & (…).

2. The three material conditional forms of the HS sequence, (<…>) → ({…}), ({…}) → ([…]) and (<…>) → ([…]) interweave with one another as a substitution-instance of the Logical Implication Form (…) ► (…).

The 'interweaving of forms with one another' is sufficient to yield *logoi* meaning as well as explain the logical form of sentences of which they are components. With the meaning of each argument component clear, the semantic validity of the *logoi* sequence can be defined as usual.

Syntactic validity is achieved once *logoi* result from uniform substitution, which they do:

(<…>) → ([…]) If <X is a rose> then [X is a flower]

([…]) → ({…}) If [X is a flower] then {X is fragrant}

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(<…>) → ({…}) If <X is a rose> then {X is fragrant}

4. A Syllogistic Logic (SL) Challenge

Can applying Plato's metaphor of 'forms interweaving with one another' yield similar insights if applied to syllogistic logic (SL)? Let's have a look.

A syntactically valid **AAA-1** structure is on left, which is an SL equivalent of HS, and on right is the result of replacing colors with *logoi* resulting from replacing antecedents and consequents above with subject and predicate terms.[35]

All ([...]) are ({...}) All [things that are flowers] are {things that are fragrant}

All (<...>) are ([...]) All <things that are roses> are [things that are flowers]

-------- --------

All (<...>) are ({...}) All <things that are roses> are {things that are fragrant}

Let us focus only meaning and validity as applied to *logoi*.

- The copula in subject position is the 'is' of identity, while the copula in predicate position is the 'is' of predication. We cannot write 'things that are identical with fragrant'.
- Forms (properties) are only expressed by terms in predicate position of a subject-predicate sentence.
- Terms in subject position do not designate Forms; they only designate objects satisfying the predicate and participate in the Form expressed by the predicate.
- In the example at hand, the Forms (properties) expressible by terms in predicate position are *being things that are fragrant* and *being things that are flowers.*
- There are no forms Forms corresponding to the two occurrences of 'being things that are (identical with) roses' occurring in subject position in the minor premise and in the conclusion; meaning that there are no Forms with which the Forms (properties) *being things that are flowers* and *being things that are fragrant*, respectively, can 'interweave.'
- As a result, the 'interweaving Forms' metaphor cannot fully account for the meaning of the *logoi* in the argument.
- As a result, the 'interweaving Forms' metaphor cannot explain the semantic validity of the *logoi* sequence.

Plato's beautiful metaphor of 'interweaving Forms' is a perfect fit with modern logic. The fact that it does not yield similar insights in SL should be considered a limitation in SL, not the metaphor. Indeed, we know from modern logic after Frege that SL is inadequate as a general theory of logic. The 15 valid syllogisms of SL, understood as rules of inference, are not adequate for the purpose of capturing the syntactic validity of mathematical proofs.

[35] Brackets here enclose SL-type terms as previously described.

5. Types and Tokens

Platonic Forms are abstract objects, meaning (in part) that they lack spatio-temporal properties. They can be related to objects that do have such properties by being exemplified by them, taken singly or in pairs. With that in mind, the 'interweaving Forms' metaphor of *Sophist*, which Plato tells us is what makes language possible,[36] brings up an interesting distinction not fully realized until C.S. Peirce. The distinction can be explained in our context by considering two ways in which Forms 'interweaving with one another' might make language, *logoi*, possible:

- Forms 'interweave with one another' to make possible *logoi* understood as concrete linguistic objects that have spatio-temporal properties such as speech acts or inscriptions on a page or computer screen; in other words, *logoi* understood as belonging in the same ontological category as other concrete objects that participate in the Forms.
- Forms 'interweave with one another' to make possible *logoi* understood as abstract linguistic objects that lack spatio-temporal properties and as such belong in the same ontological category as the Forms themselves.

The interesting distinction I have in mind, first drawn by C.S. Peirce, is between tokens and types. The first *aitia* has Forms 'interweaving with one another' to make possible token *logoi*, while the second has Forms 'interweaving with one another' to make possible type *logoi*. Put another way, under the first *aitia*, the relationship under consideration is type-token, while under the second the relationship is type-type. It is beyond the scope of this article to spell out the implications of these *aitiai* and assess their merits.

6. Is Participation Paradoxical?

British philosopher Gilbert Ryle (1900-1976) caused a great deal of controversy with the 1949 publication of *The Concept of Mind*. No less controversial was his work on Plato, including his contribution to the *Encyclopedia of Philosophy* (1967, Vol. V, 314-333) and his 1939 article on Plato's *Parmenides*.

Ryle's *Parmenides* article contains an argument claiming to show that the participation relation of Plato's Theory of Forms leads to a vicious infinite regress. Here is the passage in which Ryle states his argument (Ryle 1965, 106-107).

> Now what of the alleged relation itself, which we are calling 'exemplification'? Is this a Form or an instance of a Form? Take the two propositions 'this is square' and 'that is circular'. We have here two different cases of something exemplifying something else. We have two different instances of the relation being-an-instance-of. What is the relation between them and that of which they are instances? It will have to be exemplification Number 2. The exemplification of *P* by *S* will be an instance of exemplification, and its being in that relation to

[36] Only White 1997 and MacKay 1868 use the term 'possible'. I will be careful not to read too much into translation choices not exemplified by the other translations.

> exemplification will be an instance of second-order exemplification, and that of a third, and so on *ad infinitum*. This conclusion is impossible. So there is no such relation as being-an-instance-of. 'This is green' is not a relational proposition, and 'this is bigger than that' only mentions one relation, that of being-bigger-than.

An Imaginary Dialogue

SOCRATES: What say you to this, Phaedo and Theaetetus?
PHAEDO: Even if I grant that Ryle has proved the existence of an infinite regress of participation Forms, it does not follow that the regress is vicious. After all, orders of infinity are nothing new in mathematics. They would not be a problem for our Theory of Forms either, which is ontologically as rich as it needs to be. Ryle's argument is invalid.
SOCRATES: Excellent, Phaedo! Have you anything to add, Theaetetus?
THEAETETUS: Ryle's alleged orders of infinity collapse into one, because relational Forms such as Participation can also be One-in-Many, so that the same Participation Form can be participated in as often as needed just like any other Form. Rejecting this without argument – Ryle has none – amount to begging the question against our Theory of Forms.
SOCRATES: Bravo, Theaetetus![37]

7. One Form or Two?

An Imaginary Dialogue Continued

SOCRATES: You told Ryle, Theaetetus, that the same Participation Form can be participated in as often as needed just like any other Form.
THEAETETUS: Was I wrong about that?
SOCRATES: No, not at all. It's just that your comment brings up an important question we should talk about sooner rather than later. I'm sure it will come up eventually when philosophers question our theory, as they surely would.
THEAETETUS: What important question do you have in mind?
SOCRATES: Just this: What does it mean to say that a collection of things that have something in common participate in the same Form?
THEAETETUS: I don't see the problem. They just do, it's as simple as that.
SOCRATES: You're not seeing the point of the question, which is: What does it mean to say that we have one Form that's participate in, not two or three?
THEAETETUS: An example would help me understand what you're getting at.
SOCRATES: Consider two geometric Forms, *Equilaterality* and *Equiangularity*. A geometric figure is *equilateral* when all its sides are equal and *equiangular* when all its interior angles are equal. So, are *Equilaterality* and *Equiangularity* the same Form with different names or different Forms?

[37] Compare Cusmariu 1980.

THEAETETUS: I think geometry can answer the question. All rectangles are equiangular but only squares are both equiangular and equilateral. Therefore, the Forms *Equilaterality* and *Equiangularity* are not the same Form because something is true of *Equiangularity* – true of all rectangles – that is not true of *Equilaterality* – true only of squares.
SOCRATES: Excellent! Forms are the same because and only because what is true of one is true of the other and vice versa. Come to think of it, that's what it means for things to be the same in general, not just Forms. I hope someone figures this out eventually.
THEAETETUS: The sooner the better.
SOCRATES: Here is a related problem. The *logoi* 'equilateral' and 'equiangular' that we use to name the Forms *Equilaterality* and *Equiangularity* do not mean the same thing.
THEAETETUS: Of course not. 'Equilateral' means having equal sides and 'equiangular' means having equal angles.
SOCRATES: So, should we infer that the Forms *Equilaterality* and *Equiangularity* are not the same Form because the *logoi* 'equilateral' and 'equiangular' that we use to name them do not mean the same thing?
THEAETETUS: I hope someone figures this out eventually too.

Phaedo Sentence 7

That, I think, is the safest answer I can give myself or anyone else. [Grube 1997, 86]

- **Prolegomena**

1. Safe = Tautological?

Two major Plato scholars, G.M.A Grube and Paul Shorey, sought to interpret Plato's 'safe' comment using a concept of modern logic. I agree, of course, as did Whitehead, that Plato anticipated concepts and theories developed millennia later. Though Grube and Shorey were mistaken, their proposal is an instructive failure, as we shall see.

- Grube 1980, 19: "This would seem to be a safe answer indeed because completely tautological."
- Shorey 1933, 179: "The cause of any state or quality, as beauty, is the presence of or participation in – it makes no difference which – the idea of that state or quality. Plato is apparently aware that this in modern terms is only a tautological logic, or, as I have repeatedly put it, a consistent and systematic substitution of the logical reason for all other forms of cause. That is the primary meaning, whatever the metaphysical implications."

In short, the statement that something is beautiful if and only if it participates in Beauty is safe because it is a tautology. Well, is it?

In the dictionary sense, a tautology is a statement that 'states the same thing twice' explicitly such as 'a rose is a rose'; or implicitly such as 'flammable liquids catch fire,' which becomes an explicit tautology by replacing the phrase 'flammable liquids' with the phrase 'liquids that catch fire' according to the definition of 'flammable liquids', resulting in 'liquids that catch fire catch fire.'

Now, the statement that something is beautiful if and only if it participates in Beauty is obviously not an explicit tautology; nor is it an implicit tautology. We cannot replace 'is beautiful' with 'participates in Beauty' and vice versa using the definition of 'is beautiful' and 'participates in Beauty', respectively, as we could with 'flammable liquids.' After all, the statement that something is beautiful if and only if it participates in Beauty is entailed by a substantive philosophical theory. So, the statement that something is beautiful if and only if it participates in Beauty is not a tautology in the dictionary sense.

In logic, a statement is a tautology if and only if it is true for all possible truth-value assignments to its components. The statement

(S) X is beautiful if and only if X participates in Beauty,

is a tautology in this technical sense if and only if it is true for all possible truth-value assignments to its components,

(S1) X is beautiful

and

(S2) X participates in Beauty.

So, is S a tautology in a technical sense?

A truth table will display possible truth-value assignments to S1 and S2 and possible truth-values of their conjunction, which is what the biconditional S says.

	S1	S2	S1 → S2	S2 → S1	(S1 → S2) & (S2 → S1)
Row 1	T	T	T	T	T
Row 2	F	T	T	F	F
Row 3	T	F	F	T	F
Row 4	F	F	T	T	T

- Row 1: So far, so good.
- Row 4: It is possible that nothing is beautiful, so that a truth-value False is a possible assignment to S1. It is also possible that nothing participates in Beauty. After all, Platonic Forms do not need to be exemplified to exist, so that a truth-value False is a possible assignment to S2 as well. Thus, Row 4 is true. So far, so good here as well.
- Row 2: How is it possible for something participate in Beauty, if nothing is beautiful? We don't have a possible combination of truth values in this row, so we can rule it out.

- Row 3: Plato holds that if there are Forms, then they exist necessarily—not that they exist necessarily. From this it does not follow, however, that it is necessarily true that there are Forms, nor that there are Forms. Accordingly, Row 3 represents possible truth-value assignments to S1 and S2 because it is logically possible for something to be beautiful in the absence of Forms—ask Aristotle or any nominalist! It's just that, according to Plato, we would not have an *aitia* of what it is to be F in the absence of F-ness. So, Row 3 means that S is not a tautology in the logical sense of the term.

In conclusion, here is another intuitively valid argument whose validity SL cannot confirm:

1. A statement is a tautology either in the dictionary sense or the logical sense.

2. The statement that something is beautiful if and only if it participates in Beauty is not a tautology in the dictionary sense or the logical sense.

Therefore,

3. The statement that something is beautiful if and only if it participates in Beauty is not a tautology.

Now what?

2. Safe = Analytic?

Perhaps the statement S that something is beautiful if and only if it participates in Beauty is 'safe' can be interpreted using another modern concept, this time from Kant: S is analytic, meaning that a statement is analytic provided its negation is an inconsistent statement either explicitly or implicitly.[38]

- Explicit Inconsistency

Assuming uniform substitution for sentence letters and standard interpretation of the symbols & and ≡, an explicit inconsistency is a statement of the form p & ~p or p ≡ ~p. The statement S that something is beautiful if and only if it participates in Beauty is analytic if and only if its negation, ~S, the statement that it is not the case that something is beautiful if and only if it participates in Beauty, is a substitution instance of p & ~p or of p ≡ ~p. The statement S is evidently not a substitution instance of either form of inconsistency, from which it follows (not according to SL!) that S is not explicitly analytic.

- Implicit Inconsistency

The statement S that something is beautiful if and only if it participates in Beauty is implicitly analytic if and only if an explicit inconsistency of the form p & ~p or of the form p ≡ ~p results from its negation, ~S, the statement that it is not the

[38] As Van Cleve 1999 pointed out (18-20), Kant gives two different versions of the analytic/synthetic distinction. I will be making explicit the second version.

case that something is beautiful if and only if it participates in Beauty, by applying standard rules of logic and/or substitutions allowed by standard definitions.[39]

To decide this issue, we note first that S is a biconditional and has two components, whose negations are listed under them:

(1) If something is beautiful, then it participates in Beauty.

(1a) Something is beautiful and does not participate in Beauty.

(2) If something participates in Beauty, then it is beautiful.

(2a) Something participates in Beauty and is not beautiful.

- Case 1

To derive an explicit inconsistency of the form p & ~p from (1a), we must replace its first conjunct, the statement that something is beautiful, with the statement that something participates in Beauty. Such replacement, however, is not sanctioned by any definition of the statement that something is beautiful because (1) is part of a substantive philosophical theory, not a mere definition. It follows (though not according to SL!) that (1) is not analytic.

- Case 2

To derive an explicit inconsistency of the form p & ~p from (2a), we must replace its first conjunct, the statement that something participates in Beauty, with the statement that something is beautiful. Such replacement, however, is not sanctioned by any definition of the statement that something participates in Beauty because (2) is also part of a substantive philosophical theory, not a mere definition.

It follows that (2) is also not analytic. It follows finally that statement S that something is beautiful if and only if it participates in Beauty is neither explicitly nor implicitly analytic. Assuming that statements are either analytic of synthetic, it follows that statement S is synthetic.

3. Safe = A Priori?

Though Plato considers S

(S) X is beautiful if and only if X participates in Beauty

'safe', this is not true in the sense of 'tautology' or 'analytic'. On the other hand, he thinks B

[39] For example, a standard definition of 'is a rose' would include 'is a flower,' so that "a rose is a flower" is analytic because an explicit inconsistency is derived by replacing 'a rose' with 'is a flower' in the negation of 'if X is a rose, then X is a flower,' yielding 'it is not the case that if X is a flower, then X is a flower,' which has the form '~(if p, then p)', which logically implies 'p & ~p'.

(B) X is beautiful if and only if X is FGH ('these other reasons')

is 'confusing' and, if asked, would have considered it also 'unsafe'. Of course, B is also not a tautology, nor is it analytic. If possible, then, we must find a way to characterize the evident difference between them according to which S is 'safe', while B is 'unsafe'.

Stated in modern terms, a key difference between S and B is between *a priori* and *a posteriori* justifiability.[40] Is S 'safe' because it is *a priori*; while B is 'unsafe' because it is *a posteriori*? What is 'safe' about being a *priori* such that being a *posteriori* is 'unsafe'?

First, we need clarity about each type of justifiability.[41]

- *A Priori* Justifiability

Proposition P is justifiable *a priori* for person X =df Thinking about or understanding the meaning of P is sufficient for justifiability; empirical facts are unnecessary.

- *A Posteriori* Justifiability

Proposition P can be justified *a posteriori* for person X =df Thinking about or understanding the meaning of P is not sufficient for justifiability; empirical facts are necessary.

So, is statement S that something is beautiful if and only if it participates in Beauty, justifiable *a priori* or *a posteriori*?

Applying the definition, statement S that X is beautiful if and only if X participates in Beauty is justifiable *a posteriori* for X only if empirical facts are necessary for justifiabilty. So, what empirical facts are necessary for the two components of S to be justifiable for X?

(S1) If something is beautiful, it participates in Beauty?

(S2) If something participates in Beauty, it is beautiful.

There aren't any. It follows that S is not justifiable *a posteriori* for X. S is either justifiable *a priori* or justifiable *a posteriori*. On the other hand, because empirical facts are unnecessary for S1 and S2 to be justifiable for X and so is thinking about or understanding the meaning of S1and S2, it follows that S is justifiable *a priori* for X.

[40] Moravchik writes (1970, 56): "… distinctions like that between the a priori and the empirical are often drawn by Plato (though never exactly in such terms or terms coextensive with these)."
[41] For present purpose we only need clarity about justifiability *a priori* and *a posteriori* rather than justification as such, so that epistemologically complex issues about these concepts can be sidestepped.

Empirical facts are clearly necessary for the statement B that something is beautiful in terms of 'these other reasons' to be justfiable for X. This is true of both halves of B:

> (B1) If something is beautiful, then 'it has a bright color or shape or any other such attribute.'
>
> (B2) If something 'has a bright color or shape or any other such attribute', then it is beautiful.

Merely thinking about or understanding the meaning of B1 or B2 is clearly not sufficient for justifiability. So, B is only *a posteriori* justifiable.

The remaining issue is to understand why being a *priori* justifiable is 'safe', while being a *posteriori* justifiable is 'unsafe'. The issue is of fundamental significance.

Phaedo Sentence 8

To that I cling, in the persuasion that I shall never be overthrown. [Jowett 2023 (1892), 92]

And if I stick to this I think I shall never fall into error. [Grube 1997, 86]

- **Prolegomena**

1. Geometric Safety

It was geometric reasoning that provided Plato the confidence that it was 'safe' for him to 'cling to' the statement S that X is beautiful because and only because X participates in Beauty. He realized that he would 'never be overthrown' or 'fall into error' if he applied geometric reasoning to derive S as a theorem from his Theory of Forms; just as he knew he would 'never be overthrown' or 'fall into error' if he applied geometric reasoning to derive theorems from the axioms and definitions in geometry. Knowledge of geometry was required for admission to Plato's Academy.

In fact, the reasoning required to derive S from the Theory of Forms is much, much simpler than any reasoning exemplified in Euclid. To derive S from TF, all we would need to do is replace the predicate letter 'F' in 'X is F' with the predicate 'beautiful' and the predicate expression 'F-ness' with the predicate 'Beauty'.

> (TF) X is F because and only because X participates in F-ness.
>
> (S) X is beautiful because and only because X participates in Beauty.

However, it took millennia to devise a logic that could confirm the validity of the inference from TF to S, finally satisfying Phaedo's requirement.

2. Counterfactual Safety

An Imaginary Dialogue

SOCRATES: Consider next another kind of reason why it is safe for us to say that something is beautiful because and only because it participates in Beauty.
PHAEDO: What reason would that be?
SOCRATES: Patience, my young friend, patience. Suppose nothing was beautiful.
PHAEDO: You mean, except Beauty itself.
SOCRATES: Yes, of course, I see you've been paying attention. Now, listen, if it were to be the case that nothing was beautiful, how would anything participate in Beauty? It wouldn't.
PHAEDO: Obviously not.
SOCRATES: On the other hand, if nothing participated in Beauty, how would anything come to be beautiful? It wouldn't either.
PHAEDO: Again, obviously not.
SOCRATES: But wait, there's more.
PHAEDO: There is?
SOCRATES: If something were to be beautiful, it would participate in Beauty, right?
PHAEDO: Obviously.
SOCRATES: And if something were to participate in Beauty, it would be beautiful, right?
PHAEDO: Of course it would.
SOCRATES: Well, then, no matter how you look at it, it's safe for us to say that something is beautiful because and only because it participates in Beauty. This is true of all things of this kind.
PHAEDO: How, exactly?
SOCRATES: The Form F-ness is the cause of a thing X being F in two more senses: (a) if X wasn't F, it would not participate in F-ness and vice-versa; and (b) if X were to be F, it would participate in F-ness and vice versa. The Forms can explain what happens in actual as well as hypothetical circumstances. That's what makes them safe.
PHAEDO: What about 'those other reasons' why something is beautiful because it has a bright color or shape or some other such attribute, isn't that explanation safe too?
SOCRATES: Let's work it out together. If it were to be the case that nothing was beautiful, would it be the case that nothing had a bright color or shape or some other such attribute?
PHAEDO: Of course not! Things might well have a bright color or shape or some other such attribute even if they weren't beautiful.
SOCRATES: What if things happened to have a bright color or shape or some other such attribute, would it be the case that they were beautiful?
PHAEDO: Not necessarily.

SOCRATES: So, those other reasons are not safe because they cannot explain what happens either in actual or in hypothetical circumstances.
PHAEDO: Apparently not.[42]

3. The Ethics of Belief

Though the title of this section comes from W.K. Clifford (1877 [1999]), the idea is much, much older and in fact comes, not surprisingly, from Plato.[43] It was Plato who first pointed out, via Socrates, that in addition to moral duties, there were also (equally?) compelling what we now call doxastic duties imposed by rationality. The two main doxastic duties are

(a) to seek the truth (which Clifford does not mention)

and

(b) to avoid error (which Clifford sort of implies).[44]

Socrates carried out both duties as he applied the method named after him to a broad range of philosophically important questions.

Summing Up

This article has sought to create a new philosophical discipline, analytical history of ideas, exemplifying by linking a famous passage in the *Phaedo* with key passages in *Sophist*, *Theaetetus* and *Cratylus* to show that collectively they anticipate a significant number of important modern developments in logic, metaphysics and epistemology; and represent unity in Plato's thought at a level not heretofore realized, not even by Shorey (e.g., Shorey 1903).

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[42] Commentators on causation in *Phaedo* have missed these points.
[43] A lengthy survey article on the ethics of belief, Chignell 2018, makes no mention of Plato.
[44] Here is Clifford's second principle: "It is wrong always, everywhere, and for anyone to ignore evidence that is relevant to his beliefs, or to dismiss relevant evidence in a facile way". (Van Inwagen 1996, 145) Presumably ignoring or dismissing relevant evidence is a bad thing because it can lead to error in the form of false belief.

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