THE *COGITO* PARADOX

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Abstract: The *Cogito* formulation in *Discourse on Method* attributes properties to one conceptual category that belong to another. Correcting the error ends up defeating Descartes' response to skepticism. His own creation, the Evil Genius, is to blame.

Keywords: Descartes, *Discourse on Method*, *Cogito*, truth, certainty, validity, clear and distinct perception, skepticism, the Evil Genius.

Overview

In Discourse on Method, Descartes

(a) refers to a *Cogito* in argument form as "*cette verité*," "this truth,"

(b) describes it as "*si ferme et si assurée*," "so firm and sure,"

(c) claims it is immune to skeptical attacks on its certainty, and

(d) concludes he could accept it "without scruple as the first principle ('*le premier principe*') of the philosophy that I was seeking."

But arguments are not true or false, nor certain or uncertain; their components are. To clear up matters, the machinery of modern logic must be deployed. *Zut alors*! Disaster follows, perpetrated by Descartes' own creation, the Evil Genius: (c) is false, toppling (d) and blunting Descartes' response to skepticism.

Discourse Cogito Translations

Sources

To mitigate translation bias, I consulted eight English translations of the *Discourse*: Haldane 1970 [1911], Veitch 1912, Cottingham 1985, Cress 1998, Clarke 1999, Maclean 2006, Kennington 2007 and Bennett 2017. For the French original, I used the Amazon Kindle edition of Descartes' works, which is based on the 1874 Levrault edition.

Comments on Translations

- Haldane, Veitch, Cress, Clarke and Kennington translate "*je pense, donc je suis*" as the familiar "I think, therefore I am"; while Cottingham, Maclean and Bennett translate as "I am thinking, therefore I exist." The occurrent sense, "I am thinking," seems to me preferable to the dispositional "I think," for several reasons.

- First, the occurrent sense is implied by the use of the temporal conjunction "*pendent*," "while," in "*Mais aussitôt après je pris garde que, pendent que je voulois ainsi penser que tout étoit faux …*"
- Second, later in the same sentence Descartes writes "*il faloit nécessairement que moi qui le pensois fusse quelque chose*," whose correct translation requires the occurrent sense, namely, "it was necessarily the case that I, who was thinking …"
- Thus, when Descartes states his *Cogito* a few words later in the same sentence, using the familiar "*je pense, donc je suis*" formulation, only a translation that uses the occurrent sense, "I am thinking," is consistent with the passage as a whole.
- Descartes characterizes the *Cogito* as "*si ferme et si assurée*," which translations render in a variety of ways, e.g., "so certain and so assured" (Haldane); "so firm and sure" (Cottingham); "so firm and certain" (Clarke); and "so secure and certain" (Maclean). Descartes seems to me to use two terms, "*ferme*" and "*assurée*," for emphasis, not because epistemic value is somehow additive, or because the certainty of the *Cogito* is enhanced if both terms are used as opposed to either by itself.
- Six of the eight translations render the end of "*il faloit nécessairement que moi qui le pensois fusse quelque chose*" as "was (had to be) something." The "*quelque chose*" language means "something or other" because Descartes recognizes that the nature of the self is a separate issue. This may be why Haldane and Veitch use the awkward, though neutral locution "should be somewhat."

Cogito Passage Translations

Haldane 1970 [1911], I, 101: And since all the same thoughts and conceptions which we have while awake may also come to us in sleep, without any of them being at that time true, I resolved to assume that everything that ever entered into my mind was no more true than the illusions of my dreams. But immediately afterwards I noticed that whilst I thus wished to think all things false, it was absolutely essential that the 'I' who thought this should be somewhat, and remarking that this truth '*I think, therefore I am*' was so certain and so assured that the most extravagant suppositions brought forward by the skeptics were incapable of shaking it, I came to the conclusion that I could receive it without scruple as the first principle of the Philosophy for which I was seeking.

Veitch 1912, 42-43: Finally, when I considered that the very same thoughts (presentations) which we experience when awake may also be experienced when we are asleep, while there is at that time not one of them true, I supposed that all the objects (presentations) that had entered into my mind when awake, had in them no more truth than the illusions of my dreams. But immediately upon this I observed that, whilst I thus wished to think that all was false, it was absolutely necessary that I, who thus thought, should be somewhat; and as I observed that this truth, *I think, therefore I am*, was so certain and of such evidence that no ground of doubt, however extravagant, could be alleged by the skeptics capable of shaking it, I concluded that I might, without scruple, accept it as the first principle of the philosophy of which I was in search.

Cottingham 1985, 127: Lastly, considering that the very thoughts we have while awake may also occur while we sleep without any of them being at that time true, I resolved to pretend that all the things that had ever entered my mind were no more true than the illusions of my dreams. But immediately I noticed that while I was trying thus to think everything false, it was necessary that I, who was thinking, was something. And observing that this truth '*I am thinking, therefore I exist*' was so firm and sure that all the most extravagant suppositions of the skeptics were incapable of shaking it, I decided that I could accept it without scruple as the first principle of the philosophy I was seeking.

Cress 1998, 18: And finally, considering the fact that all the same thoughts we have when we are awake can also come to us when we are asleep, without any of them being true, I resolved to pretend that all the things that had ever entered my mind were no more true than the illusions of my dreams. But immediately afterward I noticed that, while I wanted thus to think everything was false, it necessarily had to be the case that I, who was thinking this, was something. And noticing that this truth—*I think, therefore I am*—was so firm and so assured that all the most extravagant suppositions of the skeptics were incapable of shaking it, I judged that I could accept it without scruple as the first principle of the philosophy I was seeking.

Clarke 1999, 24-25: Finally, since I thought we could have all the same thoughts, while asleep, as we have while we are awake, although none of them is true at that time, I decided to pretend that nothing that ever entered my mind was any more true than the illusions of my dreams. But I noticed, immediately afterwards, that while I thus wished to think that everything was false, it was necessarily the case that I, who was thinking this, was something. When I noticed that this truth, 'I think, therefore I am' was so firm and certain that all the most extravagant assumptions of the skeptics were unable to shake it, I judged that I could accept it without scruple as the first principle of the philosophy for which I was searching.

Maclean 2006, 28: Finally, considering that all the same thoughts which we have while awake can come to us while asleep without any one of them then being true, I resolved to pretend that everything that had ever entered my head was no more true than the illusions of my dreams. But immediately afterward I noted that, while I was trying to think of all things being false, it was necessarily the case that I, who was thinking them, had to be something; and observing this truth: *I am thinking, therefore I exist*, was so secure and certain that it could not be shaken by any of the most extravagant suppositions of the skeptics, I judged that I could accept it without scruple, as the first principle of the philosophy I was seeking.

Kennington 2007, 32-33: And finally, considering that all the same thoughts that we have while away can come to us also while we are sleeping, without there being any that are then true, I resolved to feign that all the things that had ever entered my mind were not more true than the illusions of my dreams. But immediately after, I noticed that while I thus chose to think that everything was false, it was necessarily true that I, who was thinking this, was something. And observing that this truth *I think, therefore I am* was so firm and so assured that all the most extravagant suppositions of the skeptics were incapable of shaking

> it, I judged that I could accept it without scruple as the first principle of the philosophy that I was seeking.
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> **Bennett 2017, 14-15:** Lastly, I decided to pretend that everything that had ever entered my mind was no more true than the illusions of my dreams, because all the mental states we are in while awake can also occur while we sleep and dream, without having any truth in them. But no sooner had I embarked on this project than I noticed that while I was trying in this way to think everything to be false it *had* to be the case that I, who was thinking this, was something. And observing that this truth **I am thinking, therefore I exist** was so firm and sure that not even the most extravagant suppositions of the skeptics could shake it, I decided that I could accept it without scruple as the first principle of the philosophy I was seeking.

A *Cogito* Valid in the Sentential Calculus (SC)[1]

Preliminary

Two claims made in the Overview section require explanation:

- It is a mistake to attribute semantic and epistemic properties to arguments rather than argument components.
- Descartes made this mistake in formulating the *Discourse Cogito*.

As to the first error, syllogistic logic, which Descartes studied, does not give meaning, for example, to "**AAA-1** is true" and "**AAA-2** is false."[2] Moreover, descriptions such as "**AAA-1** is justified" and "**AAA-2** is unjustified" merely note that **AAA-1** is one of sixteen syllogisms valid in syllogistic logic, while **AAA-2** is not. The sentential and quantificational calculi likewise do not define arguments as true or false, justified or unjustified. A step in an argument can be said to be justified in the sense that the correct rule of inference was applied correctly. The sixteen valid syllogisms are effectively rules of inference, though they are inadequate for validating even the simplest proofs in Euclid.

As to the second error, several passages in the *Discourse* in addition to the *Cogito* passage attribute semantic and epistemic properties to arguments rather than argument components. That said, it should be noted that there are also *Discourse* passages that attribute semantic and epistemic properties correctly to opinions, thoughts or propositions. How the literature explains (if at all) why Descartes got it right in some places and not others, I do not know. I am not about to accuse him of carelessness.

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[1] Objections will no doubt occur as readers work their way though the details below. To avoid disrupting the flow, I have relegated the matter to "Objections and Replies" at the end. I have also avoided debating the literature, which would have required book-length treatment.

[2] These are standard abbreviations in syllogistic logic. See below.

Discourse Examples of Correct Attributions

- (*Œuvres Complètes*, Kindle Edition, 20): "*… considérant combien il peut y avoir de diverses opinions* (**opinions**) *touchent une même matière*, *qui soient soutenues par des gens doctes, sans qu'il y en puisse avoir jamais plus d'une seule qui soit vraie* (**true**) …" A semantic property is attributed to opinions.
- (*Œuvres Complètes*, Kindle Edition, 36): "*… decouvrir la faussete* (**falsity**) *ou l'incertitude* (**uncertainty**) *de propositions* (**propositions**) *que j'examinais, non par des faibles conjectures* (**feeble conjectures**)*, mais par des raisonnements* (**reasonings**) *clairs et assurés* (**clear and certain**) …" The passage attributes semantic and epistemic properties to propositions and conjectures but, paradoxically, ends by attributing epistemic properties to reasonings.
- (*Œuvres Complètes*, Kindle Edition, 39): "*… des opinions* (**opinions**) *qu'en sait être fort incertaines* (**uncertain**), *tout de même que si elles étaient indubitables* (**indubitable**) …" Epistemic properties are attributed to opinions.
- (*Œuvres Complètes*, Kindle Edition, 39): "*… et enfin, considerant que toutes les memes pensees* (**thoughts**) *que nous avod etant eveilles nous peuvent aussi venire quand nous dormons, sans qu'il y en ait aucune qui soit vraie* (**not true**) …" A semantic property is attributed to thoughts: those had while asleep are said to be not true at that time.
- (*Œuvres Complètes*, Kindle Edition, 39): "*Aprés cela je considérai en général ce qui est requis à une proposition* (**proposition**) *pour être vraie et certaine* (**true and certain**) …" Semantic and epistemic properties are attributed to propositions.
- (*Œuvres Complètes*, Kindle Edition, 43): "*En sorte que si nous en avons assez souvent [idées]* (**ideas**) *qui contiennent de la fausseté* (**falsehood**)…" A semantic property is attributed to ideas. Though the term "*idées*" had to be added, it is clear that the reference is to ideas. Earlier in the passage, there is a reference to "*idées ou notions*," "ideas or notions."
- (*Œuvres Complètes*, Kindle Edition, 44): "*… ne doivent aucunement nous faire douter* (**doubt**) *de la vérité* (**truth**) *des pensées* (**thoughts**) *que nous avons éntant éveillés*." Semantic and epistemic properties are attributed to thoughts.
- (*Œuvres Complètes*, Kindle Edition, 44): "*… elle nous dicte aussi que nos pensées* (**thoughts**) *no pouvant être toutes vraies* (**true**) *…"* A semantic property is attributed to thoughts.

Discourse Examples of Incorrect Attributions

- (*Œuvres Complètes*, Kindle Edition, 19): "*Je me plaisais surtout aux mathématique, à cause de la* ***certitude*** *et de* ***l'évidence*** *de leurs* ***raisons***." Haldane, Veitch, Cottingham, Cress, Clarke Maclean, Kennington and Bennett all agree that "*à cause de la certitude et de l'évidence de leurs raisons*" attributes the epistemic terms "certainty" and "evidence" to reasoning rather than reasons, i.e., arguments rather than argument components, and translate accordingly. Some, however, translate "*raisons*" using the terms "demonstrations," "proofs" or "arguments," while others translate it literally

as "reasonings." Maclean uses "incontrovertible" instead of the literal "certain" based on interpretive comments in his Introduction (Maclean 2006, *l*.)

- (*Œuvres Complètes*, Kindle Edition, 29): *"... il n'y a eu que les seuls mathematicisens qui on pu trouver quelques* ***demonstrations****, c'est-a-dire quelques* ***raisons certaines*** *et* ***evidentes*** *...*" Cottingham, Cress, Clarke, Maclean, Kennington and Bennett agree that "*demonstrations, c'est-a-dire quelques raisons certaines et evidentes ...*" attributes the epistemic terms "certain" and "evident" to reasonings rather than reasons, i.e., arguments rather than argument components, and translate accordingly. Haldane and Veitch try to "rescue" Descartes by attributing epistemic terms to reasons rather than reasonings. Veitch: "… demonstrations, that is, any certain and evident reasons." Haldane: "… demonstrations, that is to say, producing reasons which are evident and certain."

- (*Œuvres Complètes*, Kindle Edition, 29): "*... bien que je n'en espérasse aucune autre utilité, sinon qu'elles accoutumeraint mon esprit à se repaître de* ***vérités****, et ne se contonter point de* ***fausses raisons***." Haldane, Cress, Maclean, and Kennington translate "*fausses raisons*" literally as "false reasoning(s)." The other four translators try to "rescue" Descartes in various ways from attributing a semantic property to arguments. Veitch: "such reasonings as were unsound." Cottingham and Bennett: "bad reasoning." Clarke: "faulty reasoning."

- (*Œuvres Complètes*, Kindle Edition, 39): "*… je rejetait comme* ***fausse*** *toutes les* ***raisons*** *que j'avois prise aupravant pour les* ***démonstrations*** …" Cress, Maclean, Kennington, Veitch and Clarke translate "*je rejetait comme fausse toutes les raisons*" literally as "I rejected as false all the reasoning(s) (arguments)," having translated "*demonstrations*" as "demonstrations." Cottingham and Bennett write: "I rejected as unsound all the arguments …" giving "*fausse*" the technical meaning "unsound" so it can be applied to arguments with false premises, conveniently absolving Descartes of the mistake of attributing a semantic property to arguments. Whether Descartes would have agreed that the syllogistic logic he knew recognizes the valid-sound distinction is an open question. Haldane writes: "I rejected as false all the reasons …," giving "*fausse*" its literal meaning but applying it to argument components, also absolving Descartes of the mistake of attributing a semantic property to arguments.

- Finally, the famous *Cogito* passage (*Œuvres Complètes*, Kindle Edition, 39) attributes truth and certainty to a *Cogito* stated in argument form, "*… il fallout nécessairement que moi qui le pensois fusse quelque chose; et remarquant que* ***cette vérité****,* je pense donc je suis*, étoit* ***si ferme et si assurée*** *…*" The eight translators agree that a semantic property is intended by "*cette vérité*," translated literally as "this truth." The translators also agree that an epistemic property is intended by "*si ferme et si assurée*," which they translate as follows: Haldane: "so certain and so assured." Veitch: "so certain and of such evidence." Cottingham: "so firm and sure." Cress: "so firm and so assured." Clarke: "so firm and certain." Maclean: "so secure and certain." Kennington: "so firm and so assured." Bennett: "so firm and sure."

Rescuing the *Discourse Cogito*

I will proceed as follows:

- Taking "therefore" in the *Discourse Cogito* at face value as a conclusion indicator, I will first build an argument that is valid in the sentential calculus (SC).
- Second, I will explain why this argument is not a syllogism.
- Third, I will state the valid *Cogito* argument as a sentence.
- Fourth, I will show that the resulting sentence is true—in fact, necessarily true.

As a result, Descartes avoids the following:

- misattributing truth to arguments;
- misattributing epistemic properties to arguments;
- having to say that the *Discourse Cogito* is a syllogism;
- having to say that the *Discourse Cogito* is an inference (in SC, for the time being);

Now that Descartes can refer to the *Discourse Cogito* as "*cette verité*" and "*si ferme et si assurée*" without attributing semantic and epistemic properties to arguments, the question is whether he is right that "*cette verité*" and "*si ferme et si assurée*" can be correctly be attributed to a *Discourse Cogito* in sentential form. Let us consider them in turn.

A *Cogito* Argument Valid in SC

We start by treating the two components of the *Cogito*, "I am thinking" and "I exist," as premise and conclusion and add a material conditional premise that connects them.

(1) I am thinking.

(2) If I am thinking, then I exist.

Therefore, (3) I exist.

Next, we symbolize "I am thinking" as *P*; "I exist" as *Q*; the material conditional "if, then" using the arrow symbol →; and the triangular dot symbol ∴ for "therefore."

(1*) P

(2*) $P \rightarrow Q$

∴ (3*) Q

The argument sequence from (1) and (2) to (3) is a substitution instance of the argument sequence form from (1*) and (2*) to (3*), which is the SC rule of

inference *Modus Ponens* (**MP**). Thus, the *Discourse Cogito* expressed in argument form is valid in SC.

The *Cogito* Argument Is Not a Syllogism

"Syllogism" is shorthand for "categorical syllogism in standard form," CSSF:

> (I) A CSSF is an ordered sequence of three categorical propositions in standard form, two premises and a conclusion.
>
> (II) Categorical propositions are of four types, each in subject-predicate form:
>
> **A:** All _ are _; **E:** No _ are _; **I:** Some _ are _; **O:** Some _ are not _.
>
> (III) Terms flanking the copula are of three types: A subject term, a predicate term and a middle term.
>
> (IV) Subject and predicate terms occur once in the premises and once in the conclusion.
>
> (V) The middle term occurs only in the premises in four configurations called "moods."
>
> (VI) There are a total of six term occurrences.

Inspection shows that the *Discourse Cogito* argument from (1) and (2) to (3)

> (1) I am thinking.
>
> (2) If I am thinking, then I exist.
>
> Therefore, (3) I exist.

is not a CSSF. For example:

- The components of this argument are not one of **A**, **E**, **I**, or **O**.
- Premise (2) is a material conditional and as such is not in subject-predicate form.
- Eliminating two occurrences of "I" to meet condition (VI) is not obvious.
- Reducing the *Cogito* argument to CSSF form is technically complex.[3]

[3] Anthony Kenny writes (1968: 51): "The [*Cogito*] argument could be interpreted in a simple syllogism, provided we are willing to follow Descartes in regarding 'exists' as a predicate. 'Whatever is thinking exists; but I am thinking; therefore, I exist.'" The matter is not as "simple" as Kenny thinks. Flawed accounts of a CSSF can also be found in Williams 1978, 89, Curley 1978, 27, 79), Wilson 1978, 55 and Markie 1986, 175. For Descartes' views on the syllogism, see Ariew 2011, Ch. 10. For Descartes' views on deduction see Clarke 1992, 258-285 and Gaukroger 1995, 115-118.

The *Discourse Cogito* Argument as a Sentence

To allow Descartes to refer to the *Discourse Cogito* as "*cette verité*" and "*si ferme et si assurée"* without attributing semantic and epistemic properties to arguments, we state the argument from (1) and (2) to (3) in the form of a sentence:

(1) I am thinking.

(2) If I am thinking, then I exist.

Therefore, (3) I exist.

This is easy to do because (Copi 1967, 30) to every argument there corresponds a material conditional whose antecedent is the conjunction of its premises and whose consequent is the argument's conclusion.[4] This means that we can write the argument sequence from (1) and (2) to (3) in the form of a material conditional,

(4) If I am thinking and if I am thinking, then, I exist, then, I exist,

using the conjunction "and" to make it clear that (1) and (2) are being asserted jointly. *Voila*! Descartes can now refer to the *Discourse Cogito* sentence as "*cette verité*" and "*ferme et assurée"* without misattributing semantic and epistemic properties. But, is he right to do so in both cases? Let us consider them in turn.

Does "*Cette Verité*" Apply to the *Discourse Cogito* Sentence?

We can test whether (4) is true using a standard truth table, writing the sequence form from (1*) and (2*) to (3*) as a sentence, so that (4) is a substitution instance of (5),

(5) $(P \,\&\, (P \rightarrow Q)) \rightarrow Q$

1	2	3	4	5
P	Q	$P \rightarrow Q$	$P \,\&\, (P \rightarrow Q)$	$(P \,\&\, (P \rightarrow Q)) \rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Table 1. Truth table test

[4] This statement needs to be qualified somewhat because syllogistic logic does not define the material conditional, or any other truth functions for that matter. So, for example, we cannot write **AAA-1** in syllogistic logic as "If all *M* are *P* and all *S* are *M*, then all *S* are *P*." This means that Descartes could not have solved the misattribution problem in syllogistic logic, the only kind available in his day. An aside: I have not researched the matter and cannot say whether Descartes was aware of the serious limitations of syllogistic logic for mathematical purposes; and if he was, whether he made an effort to develop an alternative. The syllogism was (famously) considered the "final word" in logic even as late as Kant.

Column 5 shows that (5) is true for any truth-functional assignments in columns 1 and 2. Because (4) is a substitution instance of (5), the *Discourse Cogito* sentence is true and can be referred to as "*cette verité*." Moreover, (5) is a tautology. On the assumption that tautologies are necessary truth, (5) is necessarily true, as are substitution instances of it.

Is the Discourse Cogito Sentence "Ferme et Assureé"?

Skepticism is an attack on the epistemic value of a proposition, not its truth value. That is, no matter what the epistemic value of proposition *P* might be as a result of "extravagant" skeptical attacks, it does not follow that *P* is false or that *P* is not necessarily true. Similarly, doubts about *P*, however persuasive, at most can lower our confidence that *P* is true. They cannot change the truth value of *P* from true to false or from necessarily true to contingently true. To avoid confusion, this part of the *Discourse Cogito* statement

> *"… et remarquant que cette vérité, je pense, donc je suis, étoit si ferme e si assurée, que toutes les plus extravagantes suppositions de sceptiques n'étoient pas capable de l'ébranler …"*

needs to be interpreted in a way that implies

> *"… et remarquant que les plus extravagantes suppositions de sceptiques n'étoient pas capable d'ébranler la certitude de cette vérité, je pense, donc je suis …"*

Descartes' claim must be interpreted to mean that no matter how "extravagant," skeptical "suppositions" cannot change ("*ebranler*," "shake") the epistemic value of the *Cogito*. At issue, then, is the certainty of the *Cogito* argument paraphrased as a sentence, (4)

> (4) If I am thinking and if I am thinking, then, I exist, then, I exist.

That is, at issue is whether (6) is true:

> (6) I am certain that if I am thinking and if I am thinking, then, I exist, then, I exist.[5]

A Doxastic Burden Principle

The weapons for defeating (6) come from Descartes' own arsenal. One is a doxastic burden principle (DBP) requiring ability to accept what is true and reject what is not true of a proposition in order for it to have an epistemic property.

[5] Descartes applies "*si ferme et si assurée*" to the entire *Cogito*. Thus, the text does not support placing the epistemic operator this way: "If I am thinking and if I am thinking, then I exist, then I am certain that I exist." The *Cogito* does not read "I am thinking, therefore I am certain that I exist." Whether it should have and if so what difference it would make are not issues that can be pursued here.

(DBP) *P* has an epistemic property for person *S* only if *S* is able to accept what is true of *P* and reject what is not true of *P*.[6]

DBP can be derived from Descartes' epistemic principle of clear and distinct perceptions:

***Principles* I.45, Latin original (AT, 22)**: *Etenim ad perceptionem, cui certum & indubitatum judicium possit inniti, non modò requiritur ut sit clara, sed etiam ut sit disticta.*

***Principles* I.45, French translation** (*Œuvres Complètes*, Kindle Edition, 735): *Car la connaissance sur laquelle on peut établir un jugement indubitable doit être non seulment clair, mais aussi distincte.*

Haldane 1970 [1911], 237: For the knowledge upon which a certain and incontrovertible judgment can be formed, should not alone be clear but also distinct.[7]

Veitch 1912, 33: For the knowledge upon which we can establish a certain and indubitable judgment must be not only clear, but also, distinct.

Cottingham 1985, 207: A perception which can serve as the basis for a certain and indubitable judgment needs to be not only clear but also distinct.

This language suggests the following:

(CDP) *P* is certain for person *S* only if *S* clearly and distinctly perceives *P*.

We can derive DPB from CDP in a few easy steps:

(a) *S* clearly and distinctly perceives *P* only if *S* accepts *P*.

(b) *S* accepts *P* only if *S* can accept *P*.

(c) *S* can accept *P* only if *S* is able to accept what is true of *P* and reject what is not true of *P*.

Therefore, (d) *P* is certain for *S* only if *S* is able to accept what is true of *P* and reject what is not true of *P*.

We can reasonably generalize (d) to epistemic properties as such:

(DBP) *P* has an epistemic property[8] for person *S* only if *S* is able to accept what is true of *P* and reject what is not true of *P*.

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[6] *P* can be a proposition, sentence, statement and even a thought, opinion or judgment in the occurrent sense. The bearers of epistemic properties for Descartes is too complex an issue discuss in this article.

[7] Haldane states (202) that her translation "… is made from the Latin version collated with the French," which is why I'm including the French translation of the Latin original.

[8] My arguments in SC and QC rely on certainty in the epistemic sense in both CDP and DBP. For Descartes, metaphysical certainty must also be considered. For a comment on metaphysical certainty, see Objection 5 below.

Applying DBP

Inspection shows that I-VII below are true of (4)

(4) If I am thinking and if I am thinking, then, I exist, then, I exist,

while their negations are not. The list can easily be expanded.

I. The two occurrences of "if" are synonymous.[9]

II. The first occurrence of "if" pairs up with the second rather than the first occurrence of "then."

III. The two occurrences of "am" are synonymous.

IV. The two occurrences of "thinking" are synonymous.

V. The two occurrences of "then" are synonymous.

VI. The two occurrences of "exist" are synonymous.

VII. The four occurrences of "I" are co-referential.

Applying DBP yields that (4) has an epistemic property for *S* only if *S* is able to accept I-VII, which are true of (4), and reject the negations of I-VII, which are not true of (4).

Descartes' Evil Genius

The other weapon for defeating the certainty of the *Discourse Cogito* is Descartes' Evil Genius (EG), characterized at the end of *Meditation I*:

> **Heffernan 1990, 96**: *Supponam igitur non optimum Deum, fontem veritatis, sed genium aliquem malignum, eundemque summe potentem & callidum, omnem suam industriam in eo posuisse, ut me falleret.*
>
> **Cress 1979, 16**: Thus I will suppose not a supremely good God, the source of truth, but rather an evil genius, as clever and deceitful as he is powerful, who has directed his entire effort to misleading me.

A powerful and deceitful EG presumably can bring about the following states of affairs:

(i) a person accepting what is not true;

(ii) a person rejecting what is true;

(iii) a person being unable to accept what is true;

(iv) a person being unable to reject what is not true.

[9] This article is not the place to address Quine's well-known concerns about synonymy (Quine 1950.) See Cusmariu 1978B, Cusmariu 1982 and Cusmariu 1983.

For purposes of my argument, it is sufficient that the EG be able to bring about states of affairs (iii) and (iv), which need not involve causing doubt (of any kind) in a person.

Three Scenarios

The list of I-VII represents opportunities that the EG can use to cause *S* to run afoul of the doxastic burden principle DBP. An EG who has the power to impact a person's mental life in such a way as to bring about states of affairs (iii) and (iv), presumably has the power to impact a person's mental life in such a way as to cause a person to experience memory lapses while uttering or thinking (4) to himself,

> (4) If I am thinking and if I am thinking, then, I exist, then, I exist.

Here are four memory-lapse scenarios under states of affairs (iii) and (iv):

> **Scenario 1**: *S* utters or thinks to himself "if" at the beginning of (4), then continues with "I am thinking and" but just as he is about to utter or think to himself "if" as the next word after "and," the EG immediately causes *S* to experience a memory lapse so that when *S* utters or thinks to himself the "if" after "and," *S* is unable to remember that this "if" is synonymous with the "if" he already uttered or thought to himself, and as a result *S* is unable (iii) to accept what is true of (4), that the two occurrences of "if" are synonymous; and (iv) to reject what is not true of (4), that the two occurrences of "if" are not synonymous.
>
> **Scenario 2**: *S* utters or thinks to himself the first occurrences of "am," "thinking," "then," and "exist" in (4) but then the EG immediately causes *S* to experience memory lapses so that when *S* comes to the point of uttering or thinking to himself the second occurrences of these terms, *S* is unable to remember that they are synonymous with their first occurrences he already uttered or thought to himself, and as a result *S* is unable (iii) to accept what is true of (4), that the two occurrences of each of these terms are synonymous; and (iv) to reject what is not true of (4), that the two occurrences of these terms are not synonymous.
>
> **Scenario 3**: *S* utters or thinks to himself "If I" at the beginning of (4), realizes that "I" refers to himself, then *S* continues with "am thinking and if" but just as he is about to utter or think to himself "I am," the EG immediately causes *S* to experience a memory lapse so that when *S* utters or thinks to himself "I am," *S* is unable to remember that this second "I" is co-referential with the one he already uttered or thought to himself. The EG repeats this process for the remaining two occurrences of "I," and as a result *S* is unable (iii) to accept what is true of (4), that the four occurrences of "I" refer to himself; and (iv) to reject what is not true of (4), that the four occurrences of "I" do not refer to himself.[10]

[10] There are two basic approaches to the semantics of indexicals: *utterance-based* (Reichenbach 1947, Burks 1949) and *expression-based* (Kaplan 1989)—see Georgi in the *Internet Encyclopedia of Philosophy*. My use of "utterance" language does not mean I am relying on a specific analysis of either kind. I am merely following Descartes' lead in running my argument in terms of utterances. Thus, in *Meditation II* Descartes writes (Heffernan Bilingual Edition

The doxastic burden principle DBP together with inability to accept what is true and reject what is not true of (4)

(4) If I am thinking and if I am thinking, then I exist, then I exist,

imply that (6)

(6) I am certain that if I am thinking and if I am thinking, then I exist, then I exist,

is false. The *Cogito* is valid in SC, the argument can be paraphrased as a true sentence so that "*cette verité*" can be asserted, but the certainty Descartes claimed is shot down by flak from his own creation, the Evil Genius, with a key assist from a principle derived from Descartes' epistemic principle of clear and distinct perceptions.

A *Cogito* Valid in the Quantificational Calculus (QC)[11]

Preliminary

It may well be objected that a premise needed for a valid derivation in SC, "If I am thinking, then I exist," could lead to a sound argument only if this premise is true in the general case. To answer this objection, a universal material conditional linking occurrent thought to the thinker's existence must be added to the argument.

(D) Anything that is thinking, exists.

The deductive resources of SC, however, are not adequate to yield a valid inference from a premise in universal form to a conclusion in singular form. To bring about such an inference, a quantificational rule is necessary, specifically, Universal Instantiation (**UI**). We start with an explanation of **UI –** what it is, how it works, and what is involved in its correct application.

1990: 100) "… Ego sum, ego existo, *quoties a me profertur* [as it is uttered by me], *vel mente concipitur* [or conceived by the mind] *necessario esse verum*." The distinction between utterance-based and expression-based semantics matters to the third scenario but not the first two. In any case, the EG should have no difficulty defeating an expression-based indexical theory.

[11] The term "quantificational calculus" (QC) seems to me preferable to "quantification theory" (it's not a theory); "predicate logic" and "predicate calculus" (it's not just about predicates); or "functional calculus" (functions already have a meaning in mathematics.) See LeBlanc 1966: 83. A recent historical overview of QC is Ferreiros 2001. For the sake of uncluttered text, quotation marks have been kept to a minimum and no use has been made of Quine corner quotes (Quine 1940).

A *Cogito* Valid in QC

The conclusion of the *Cogito* is supposed to follow by **UI** and **MP**:

(1) (x)(If x is thinking, then x exists)	Translation of (D) in QC
∴ (2) If I am thinking, then I exist.	1, **UI**
(3) I am thinking.	Assumption
∴ (4) I exist.	2, 3 **MP**

UI in QC

The deductive resources of QC include rules of inference from SC such as **MP** and four rules governing universal and existential quantification. Here are **UI** details:

> **Formalism**: $(x)\Phi x$ / ∴ Φa
>
> **Interpretation**: From a closed universally quantified wff of the form $(x)\Phi x$, infer a singular closed wff of the form Φa.
>
> **Application Sequence**: It will be useful to present the application of **UI** to a specific case as a sequence of steps.[12]

Step 1: Determine that the wff to be used as the premise in a potential **UI** argument is a closed universally quantified wff of the form $(x)\Phi x$.
Step 2: Delete the (objectual) universal quantifier binding occurrences of the individual variable x in the closed universally quantified premise of the form $(x)\Phi x$ to obtain an open wff of the form Φx.
Step 3: Write a closed wff of the form Φa from Φx of Step 2 such that:

> (i) a in Φa is an individual constant in the signature of QC;
>
> (ii) occurrences of a in Φa are of the same individual constant;
>
> (iii) occurrences of a in Φa replace occurrences of the same individual variable x in Φx;
>
> (iv) occurrences of a are in Φa at exactly those places where occurrences of x are in Φx;
>
> (v) reference of occurrences of a in Φa has been fixed so they designate the same object in the universe of discourse (domain of quantification) of QC. [13]

Step 4: Enter ϕa on a line below $(x)\Phi x$.

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[12] Application steps are my own formulation.
[13] Individual variables and individual constants are non-logical symbols in the signature of QC, as opposed to logical symbols such as material implication and negation. This article is not the place for a precise characterization of the distinction between logical and non-logical symbols, regarding which Tarski wrote (1983, 419) "… no objective grounds are known to me which permit us to draw a sharp boundary between the two groups of terms." See also Peacocke 1976.

Comments on UI Application Steps

- The terms "individual variable" and "individual constant" are standard in logic textbooks, e.g., Hilbert and Ackermann 1950 (1928), 102[14]; Kleene 1950, 436; Carnap 1956, 4; Church 1956, 168; LeBlanc 1966, 83, 84, 92; Schoenfield 1967, 10, 12; Copi 1967, 278-9; Thomason 1970, 209; Hunter 1971, 137; Takeuti 1975, 192; Curry 1977, 316; Mendelson 1979, 46; Kalish, Montague and Mar 1980, 440-1; Simco and James 1983, 122-3; Enderton 2001, 70); Hurley 2008, 407-409, 416 [15]; and Smullyan 2014, 137.[16]
- Step 3i captures a basic syntactic fact about the signature of QC: An individual constant must come from the signature of QC to be used in a quantificational rule.
- The reader can easily verify that validity requires Steps 3ii-3iv.
- This is also true of Step 3v. Without this step, the possibility is left open that $(x)(\mathrm{F}x \rightarrow \mathrm{G}x)$ is true while a substitution instance $\mathrm{F}a \rightarrow \mathrm{G}a$ is false because the reference of the two occurrences of the individual constant a has not been fixed to denote the same object, rendering the inference of $\mathrm{F}a \rightarrow \mathrm{G}a$ from $(x)(\mathrm{F}x \rightarrow \mathrm{G}x)$ invalid. This could happen even though Steps 3ii-3iv are satisfied.
- Step 3v shows that **UI** cannot be formulated in purely syntactic terms. What this means and whether the same is true of the other three quantificational rules of QC are technical questions beyond the scope of this article.

What Is an Individual Constant?

Here are points relevant to the *Discourse Cogito* as a derivation sanctioned by **UI**.

- Symbolized using lower-case letters from the beginning of the alphabet a, b, c … individual constants enable us to express in symbols that an object in the universe of discourse of a formal theory satisfies an open wff of the form Φx, which can be expressed by writing a closed wff of the form Φa.

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[14] In Editor's Notes, Luce writes (Hilbert and Ackerman 1950, 168): "It is not always understood that a constant, like a variable, is a *symbol*, a linguistic expression, but with the important distinction that a constant has a fixed designation, which remains unaltered throughout the discussion in which the constant appears; whereas a variable designates ambiguously, so to speak, assuming any one of a range of values." The main text where the terms "constant" and "variable" occur does not contain these explanations.

[15] Hurley also uses "instantial letter," which he explains in the Glossary (677) as "the letter (variable or constant) introduced by universal instantiation or existential instantiation." The main text does not include this explanation. Presumably "instantial letter" covers both individual variables and constants, so that a better term would have been "instantial symbol."

[16] While the term "variable" is as old as algebra and mathematics generally distinguishes between "real variables" ("free variables" in logic) and "apparent variables" ("bound variables" in logic), the term "individual variable" is from logic. Mathematics does not use the term "constant," speaking instead of "values" of a variable. Construing individual constants as "values" of individual variables, as symbols denoting objects in the universe of discourse of QC is hardly a definition of "individual constant."

- Individual constants enable valid derivations of, for example, wffs about objects taken singly, Φa, from wffs about objects taken collectively, $(x)\Phi x$. All axioms and some theorems are in universally quantified form.
- Individual constants are needed to express property instantiation, e.g., instantiation of *humanity* and *mortality* in an inference from a sentence of the form $(x)(Fx \rightarrow Gx)$ such as "Whatever is human, is mortal" to a sentence of the form $Fa \rightarrow Ga$ such as "If Socrates is human, then Socrates is mortal."
- By way of definition of what it means for occurrences of an individual constant to be occurrences of the same individual constant—**UI** application Step 3ii—we will stipulate that this is true if and only if individual constant occurrences replace or are replaced by the same individual variables. This is not intended as a general definition, however, because occurrences of an individual constant can be occurrences of the same individual constant for reasons unrelated to substitutivity of individual variables or any other connection to rules of quantification.
- We will stipulate that individual variable occurrences in Ψ are occurrences of the same individual variable—**UI** application Step 3iii—if and only if the same quantifier binds all individual variable occurrences if Ψ is closed or would bind them all if Ψ is open.
- Individual constants in QC are "referentially anonymous" in the sense that they only designate some object or other in the universe of discourse of QC, to mirror the fact that the existential quantifier likewise asserts the existence only of some object or other in the universe of discourse of QC.
- Once referentially anonymous individual constants are replaced with referentially transparent constants such as numerals and proper names, application Step 3v of **UI** applies. The reference of occurrences of such individual constants needs to be fixed so they consistently designate the same object in the universe of discourse of QC. How this can be done will be discussed below.

First Preliminary Application

A **UI** derivation in arithmetic will help identify issues relevant to the *Cogito* argument.

Properties of addition, subtraction and equality allow us to write:

(A) $(x)(y)(z)(x + y = z \rightarrow x = z - y)$

UI application instructions should enable us to prove that (A) logically implies (C):

(C) $(7 + 5 = 12) \rightarrow (7 = 12 - 5)$

Step 1: Determine that the wff to be used as the premise in a potential **UI** argument is a closed universally quantified wff of the form $(x)\Phi x$.

QC does not recognize (A) as a wff because the symbols for addition and subtraction are not in its signature. For the time being, however, let us stipulate that these symbols have been added to the signature of QC so that a universally

quantified sentence of arithmetic such as (A) can be considered a wff of the form $(x)\Phi x$ in QC.

Step 1 may be understood broadly to include (A) as wff of the form $(x)\Phi x$ because the following assumptions are reasonable:

- The material conditional arrow; quantifiers; individual variables and their occurrences; left and right hand brackets have the same meaning in (A) that they have in QC.
- Individual variable occurrences are bound by quantifiers in (A) exactly as they are in QC.

Step 2: Delete the (objectual) universal quantifier binding occurrences of the individual variable x in the close universally quantified premise of the form $(x)\Phi x$ to obtain an open wff of the form Φx.

Having stipulated that (A) is a wff of the form $(x)\Phi x$, Step 2 can be applied to (A), resulting in (B):

(A) $(x)(y)(z)((x + y = z) \rightarrow (x = z - y))$

$\therefore$ (B) $(x + y = z \rightarrow x = z - y)$ 1, **UI** Step 2

Next, we decide whether Step 3 allows the argument to move from (B) to (C):

(B) $(x + y = z) \rightarrow (x = z - y)$

$\therefore$ (C) $(7 + 5 = 12) \rightarrow (7 = 12 - 5)$

Step 3: Write a closed wff of the form Φa from Φx of Step 2 such that:

(i) a in Φa is any individual constant in the signature of QC;

(ii) occurrences of a in Φa are of the same individual constant;

(iii) occurrences of a in Φa replaces occurrences of the same individual variable x in Φx;

(iv) occurrences of a are in Φa at exactly those places where occurrences of x are in Φx;

(v) reference of occurrences of a in Φa has been fixed so they designate the same individual in the universe of discourse (domain of quantification) of QC.

However, Step 3 can be applied to (B) to yield (C) provided:

(i) It can be explained how numerals can be considered individual constants in the signature of QC. For the time being, let us stipulate that the signature of QC has been expanded so as to include numerals as individual symbols. Let us stipulate further that integers have been added to the universe of discourse of QC.

(ii) It can be reasonably stipulated that the two occurrences of the numerals 7, 5 and 12 in (C) are occurrences of the same numerals. This can be done on grounds that they replace occurrences of the same respective individual variables.

(iii) It can also be reasonably stipulated that occurrences of the numerals 7, 5 and 12 in (C) replaced occurrences of the same individual variables *x*, *y* and *z*, respectively, in (B). This can be done on grounds that each variable is bound by the same universal quantifier.

(iv) It can be seen by inspection that (C) contains occurrences of 7, 5 and 12 at exactly those places where (B) contains occurrences of *x*, *y* and *z*, respectively.

(v) Finally, it can reasonably be stipulated that the reference of occurrences of 7, 5 and 12 in (C) has been fixed so that they are co-referential. How exactly this is done need not be spelled out for present purposes.

All five conditions of Step 3 having been met, we can write

$(x)(y)(z)((x + y = z) \rightarrow (x = z - y))$ / $\therefore (7 + 5 = 12) \rightarrow (7 = 12 - 5)$[17]

Second Preliminary Application

Frege and his followers built modern logic to provide a rigorous foundation for mathematical reasoning, which is why the above application of **UI** was straightforward. However, when Russell used his expansion of the uniqueness quantifier in $\exists!\Phi x$ to formulate his theory of descriptions (Russell 1905) and then apply it to "puzzles"[18] (Russell 1956, 47), a precedent was set that analytic philosophy has followed ever since. The methods of formal logic have been used to help elucidate philosophical problems despite significant differences between formal languages and the vernacular, the language in which philosophical problems are stated. Ramsey agreed, describing what started life as notational convenience as a "paradigm of philosophy" (Ramsey 1965, 263). In his *Tractatus* (1963 [1921]), Wittgenstein applied the lessons of the theory of descriptions to philosophy as a whole. In a way, the *Tractatus* is the theory of descriptions "on steroids."

As a test case closer to the *Discourse Cogito*, let us look at an argument that is intuitively valid and see what is involved in applying **UI** to confirm validity.

(A) (x)(x is human $\rightarrow$ x is mortal)

$\therefore$ (B) Socrates is human $\rightarrow$ Socrates is mortal

According to the standard understanding of **UI**, if (A) is true, it follows that every substitution instance of (A) that has the requisite form will be true. So, is (B) a substitution instance of (A), but does it have the requisite form? We follow application steps as specified above.

[17] This sentence can be converted into a material conditional but there is no need to do that.
[18] Omitted from the list of "puzzles" Russell claimed his theory of descriptions (TD) solves is his own paradox. TD allowed Russell to formulate his "no class" solution to the paradox; but then he realized there is also a version of the paradox for what he called "propositional functions, and it was back to square one; the theory of types followed.

Step 1: Determine that the wff to be used as the premise in a potential **UI** argument is a closed universally quantified wff of the form $(x)\Phi x$.

QC does not recognize (A) as a wff. (A) can be regarded as a substitution instance of a closed universally quantified sentence of the form $(x)\Phi x$ that is recognized as a wff in QC,

(C) $(x)(Fx \rightarrow Gx)$,

provided

(i) the universal quantifier in (A) functions the way it functions in (C), as do individual variables;

(ii) the material implication arrow captured the ordinary meaning of "if … then …" that makes vernacular versions of (A) true;

(iii) the predicates "is human" and "is mortal" in (A) can be considered substitution instances of "*F*" and "*G*," respectively, in (C).

For the sake of argument, let us stipulate that all these conditions have been met. We will leave it to a Substitution Theorem [19] to spell out the precise relationship between (C) and (A) and move on to application Step 2 of **UI**.

Step 2: Delete the (objectual) universal quantifier binding occurrences of the individual variable *x* in the close universally quantified premise of the form $(x)\Phi x$ to obtain an open wff of the form Φx.

Having agreed that (A) can be regarded as a substitution instance of a wff recognized as such in QC, Step 2 can be applied to (A), resulting in (A1):

(A) (x)(*x* is human → *x* is mortal)

∴ (A1) *x* is human → *x* is mortal Step 2 of **UI**

Let us move on to Step 3 of **UI** to decide the validity of this inference:

(A1) *x* is human → *x* is mortal

∴ (B) Socrates is human → Socrates is mortal

Step 3: Write a closed wff of the form Φa from Φx of Step 2 such that:

(i) *a* in Φa is any individual constant in the signature of QC;

(ii) occurrences of *a* in Φa are of the same individual constant;

(iii) occurrences of *a* in Φa replaces occurrences of the same individual variable *x* in Φx;

[19] For the Substitution Theorem, see Kleene 1967: 14. For rules of substitution, see LeBlanc 1966: 137. The idea of a rule of substitution comes from Frege. See Zalta 2018A and 2018B. Gödel has stated (Schilpp 1944, 126): "In *Principia*, eliminations are always carried out by substitutions in the theorems corresponding to definitions, so that it is chiefly the rule of substitution which would have to be proved."

(iv) occurrences of *a* are in Φ*a* at exactly those places where occurrences of *x* are in Φ*x*;

(v) reference of occurrences of *a* in Φ*a* has been fixed so they designate the same object in the universe of discourse (domain of quantification) of QC.

However, applying Step 3 to (A1) will result in (B) provided:

(I) It can be explained how proper names can be considered individual constants in the signature of QC.

(II) We can reasonably stipulate that the two occurrences of "Socrates" in (B) are occurrences of the same proper name. See below.

(III) We can reasonable stipulate that the two occurrences of "Socrates" in (B) replaced occurrences of the same individual variable *x* in (A1). See below.

(IV) It can be shown that (B) contains occurrences of "Socrates" at exactly those places where (A1) contains occurrences of *x*. Inspection shows this is the case.

(V) The reference of the two "Socrates" occurrences of (B) has been fixed so that they are co-referential.

Ordinary Proper Names as Individual Constants

Ordinary proper names (OPNs) can be included as individual constants in a QC signature based on the designative function they already have in natural language. There are three questions that must be answered.

Question 1: How do OPNs acquire a designative function?

Answer: Reference fixing for OPNs uses one or more definite descriptions to assign designation. A definite description is an open sentence of the form "the *x* such that *Fx*," which entails uniqueness according to Russell's analysis (Russell 1905), formalized in *Principia Mathematica* (Russell and Whitehead 1911, 31-2). Thus, under the interpretation of "Socrates" as "the name of the protagonist of Plato's dialogues," the sentence "Socrates is human" is true. On the other hand, under the interpretation of "Socrates" as "the name of the shaggy mutt down the street," the sentence "Socrates is human" is false. Under the interpretation of "Socrates" as "the name formed by letters S-o-c-r-a-t-e-s of the English alphabet," the sentence "Socrates is human" is either false, nonsense or lacks a truth value, depending on one's theory.

An OPN designates the object satisfying the definite description. For example, if an interpretation assigns the definite description "the author of *Waverly*" to the OPN "Scott" in a universe of discourse that includes persons, then, the OPN "Scott" is stipulated to designate the object satisfying the define description "the author of *Waverly*." Thus, it is true under this interpretation that Scott is identical with the author of *Waverly* and false that Scott is identical with the author of *Hamlet*.

Question 2: Must the designation of each OPN occurrence be fixed separately?

Answer: No. Consider as an example "If Socrates is human, then Socrates is mortal." In order for this sentence to be true, it is sufficient that the same interpretation be assigned to the two occurrences of "Socrates," which is what happens in ordinary contexts of use.

Question 3: How can the reference of OPN occurrences be fixed so they are co-referential.

Answer: The reference of OPN occurrences can be fixed so they are co-referential if and only if the same interpretation, e.g., definite description, determines the identity of the object to which they refer. Note that from the fact that two OPN occurrences are of the same OPN, it does not follow that they are co-referential. In one and the same true sentence, "Socrates" might denote the Greek philosopher and the shaggy dog down the street its owners happened to name after the Greek philosopher.

Having satisfied all conditions of Step 3, we can now write

$(x)(x \text{ is human} \rightarrow x \text{ is mortal}) / \therefore$ Socrates is human → Socrates is mortal[20]

Applying UI to the *Cogito*

Here is our *Cogito* argument again:

(D) Anything that is thinking, exists.

(1) $(x)(x$ is thinking → x exists) Translation of (D)

$\therefore$ (2) I am thinking → I exist. 1, **UI**

(3) I am thinking. Assumption

$\therefore$ (4) I exist. 2, 3 **MP**

Let us follow **UI** application instructions one step at a time.

Step 1: Determine that the wff to be used as the premise in a potential **UI** argument is a closed universally quantified wff of the form $(x)\Phi x$.

QC does not recognize (1) as a wff. The issue is whether (1) can be included in our **UI** argument as a substitution instance of a closed universally quantified sentence of the form $(x)\Phi x$ that is recognized as a wff in QC, namely,

(C) $(x)(Fx \rightarrow Gx)$.

Application Step 1 of **UI** can be applied to (1) understood as a substitution instance of (C) provided familiar conditions are satisfied:

(i) the universal quantifier in (1) functions the way it functions in (C), as do individual variables;

(ii) the material implication arrow captures the ordinary meaning of "if … then …" that makes (1) true and (1) as a correct translation of (D);

[20] There is also no need to convert this sentence into a material conditional.

(iii) the predicates "is thinking" and "exists" in (1) can be considered substitution instances of "*F*" and "*G*," respectively, in (C).[21]

We will leave it to a Substitution Theorem again to spell out the precise relationship between (C) and (1) and move on to the Step 2 of **UI**.

Step 2: Delete the (objectual) universal quantifier binding occurrences of the individual variable *x* in the close universally quantified premise of the form $(x)\Phi x$ to obtain an open wff of the form Φx.

Let us stipulate that (1) is a substitution instance of a wff of the form $(x)\Phi x$ recognized as such in QC, namely, (C), so Step 2 can be applied to (1), resulting in (1a):

(1) (x)(*x* is thinking → *x* exists)

∴ (1a) *x* is thinking → *x* exists 1, **UI** Step 2

Next, we decide whether Step 3 allows the *Cogito* argument to move from (1a) to (2):

(1a) *x* is thinking → *x* exists

∴ (2) I am thinking → I exist.

Step 3: Write a closed wff of the form Φa from Φx of Step 2 such that:

(i) *a* in Φa is any individual constant in the signature of QC;

(ii) occurrences of *a* in Φa are of the same individual constant;

(iii) occurrences of *a* in Φa replaces occurrences of the same individual variable *x* in Φx;

(iv) occurrences of *a* are in Φa at exactly those places where occurrences of *x* are in Φx;

(v) reference of occurrences of *a* in Φa has been fixed so they designate the same object in the universe of discourse (domain of quantification) of QC.

However, Step 3 can be applied to (1a) to yield (2) provided:

(i) It can be explained how the indexical "I" can be considered an individual constant in the signature of QC, which is not the case at the moment.

[21] Kant famously objected that existence was not a "real predicate," though he probably meant "property." Either way, I will not raise this issue to question a *Cogito* derivation in QC. That said, we should not confuse, as Kant seems to be doing, property existence with property exemplification. Under Platonism, a property need not be a property of anything, a view Aristotle denied. My work on property ontology stems from my Ph.D. dissertation, Cusmariu 1977. See Cusmariu 1978A, B, and C; Cusmariu 1979A and B; Cusmariu 1980; Cusmariu 1985; and Cusmariu 2016C. Platonism has had a significant influence on my artwork as a sculptor. See Cusmariu 2009; Cusmariu 2015A and B; and Cusmariu 2017A and B.

(ii) We can reasonably stipulate that the two occurrences of "I" in (2) are occurrences of the same indexical. This can be done on the familiar grounds that they replace occurrences of the same individual variable, *x*, in (1).

(iii) We can also reasonably stipulate that occurrences of "I" in (2) replaced occurrences of the same individual variable *x* in (1a). This can be done on the familiar grounds that these individual variable occurrences are bound by the same quantifier.

(iv) We can show that (2) contains occurrences of "I" at exactly those places where (1a) contains occurrences of *x*. Inspection shows that this is the case.

(v) It can be explained how the reference of the two "I" occurrences in (2) can be fixed so that they are co-referential.

Is "I" an Individual Constant In QC?

The answer is obviously "no" for a purely technical reason: There is no list of individual constants in any QC signatures that looks like this: *a*, *b*, *I*, *me*, *my*, *mine*, *c*, *d*, …

Philosophically speaking, the absence of "I" and its cognates is not surprising:

- Frege's decisively refuted psychologism, rejecting the view that truths of logic and mathematics are about, or depend upon occurrent or dispositional mental states of persons. Here Frege was in agreement with Plato and Aristotle as well as many later logicians.[22]
- Persons and their mental states are not in the universe of discourse of formal systems.[23] Set theory, for example, would have no use for "I" and its cognates ("me," "my," "mine") as individual constants, there being nothing for them to designate, not even the empty set—already symbolized by {} or Ø.

[22] In a 1918 essay titled "The Thought: A Logical Inquiry," Frege remarked (Klemke 1968, 517): "The same utterance containing the word 'I' will express different thoughts in the mouths of different men, of which some may be true, others false. The occurrence of the word 'I' in a sentence gives rise to some questions." Unfortunately, Frege did not go on to consider whether "I" is or could be an individual constant and if so under what conditions. Frege also did not explore connections between his view of "I" and the *Cogito*. The focus of his logic treatises of decades earlier was entirely different, so the issues did not arise.

[23] The only exception of which I am aware is my own work. In Cusmariu 2012 and Cusmariu 2016, I introduce semantic evidence predicates into the metalanguage of science and mathematics, which means that persons and their beliefs are included in the universe of discourse of these disciplines. However, this is consistent with Frege's critique of psychologism. Neither the truth value nor the epistemic value of scientific and mathematical beliefs is in any sense "subjective."

"I" As an Individual Constant

First-person indexicals (FPIs) such as "I," "me," "my" and "mine" can be included as individual constants in a QC signature under the same conditions that allow ordinary proper names to be included. The same three questions apply.
Question 1: How do FPIs acquire a designative function in QC?
Answer: It is sufficient for present purposes to stipulate that FPIs have a designative function in QC if and only if there is a definite description of form "the *x* such that *Fx*" such that an FPI designates the object that satisfies "the *x* such that *Fx*." What this definite description might be is context dependent, as many have pointed out, so it can be left to the context to specify it. For example, for *Discourse Cogito* purposes, this definite description can have the form "the person uttering or thinking sentence *P* to himself." Readers may pick whatever analysis of indexicals they wish.[24]
Question 2: Must the reference of each FPI occurrence of be fixed separately?
Answer: No. One and the same definite description can be used to fix the reference of all FPI occurrences.
Question 3: How can the reference of FPI occurrences be fixed so they are co-referential?
Answer: Provided the same definite description is used each time in the context at hand, co-referentiality will follow as a matter of course.

Thus, we can sidestep worries such as whether FPIs have sense as well as reference; and if they have both, what relations hold between them; whether they are directly or indirectly referential (Kaplan 1989, 523); and so on.

The QC Component of the *Discourse Cogito* as a Sentence

(1) (x)(x is thinking → x exists) Assumption

∴ (2) I am thinking → I exist. 1, **UI**

We start by writing first (1) ∴ (2) horizontally as one sentence:

(3) (x)(x is thinking → x exists) / ∴ I am thinking → I exist.

A material conditional also corresponds to the argument in (3), whose antecedent is the premise of the argument, (1), and whose consequent is the conclusion of the argument, (2), so we can write (4)

(4) (x)(x is thinking → x exists) → (I am thinking → I exist).

[24] See, for example, Reichenbach 1947; Burks 1949; Castaneda 1966; Kamp 1971; Kaplan 1989; García-Carpintero 1998; King 2001; Perry 2001; Salmon 2005; Soames 2005; and Cappelen and Dever 2013. Rosenkrantz 1993 discusses the broader metaphysical issues involved.

Syllogistic Form Is Inadequate

Descartes would have been able to recognize (4) only if stated in syllogistic form. To see if this is possible, we first do away with quantificational notation and the material conditional in the antecedent of (4) and write the result as an **A**-proposition,

> (4a) All persons who are thinking are persons who exist.

Next, we do away with the material conditional in the consequent of (4) and shoehorn the rest into an **A**-proposition as well,

> (4b) All persons identical with me who are thinking are persons identical with me who exist.

However, the two truth-functional components of (4) are connected by means of a material conditional. Replacing it with a copula to connect (4a) and (4b) results in gibberish, as the reader can readily see, proving the inability of syllogistic logic to capture anything more complicated than subject-predicate sentences. Syllogistic logic is inadequate for mathematical purposes in the sense that the sixteen valid syllogisms (rules) of syllogistic logic are insufficient to allow mathematical proofs to go through.[25]

Eliminating Quantificational Notation

An adequate paraphrase of (4) must capture its three material conditionals:

> (5) If it is true that if anyone who is thinking, then that person exists, then, it is true that if I am thinking, then I exist.

Applying "*ferme et assurée*" to (5) yields (6):

> (6) It is certain for a person that if it is true that if anyone who is thinking, then that person exists, then, it is true that, if I am thinking, then I exist.

A Doxastic Burden Principle

The doxastic burden principle, DBP,

> (DBP) *P* has an epistemic property for person *S* only if *S* is able to accept what is true of *P* and reject what is not true of *P*.

can be applied to show that (6) is false. Inspection shows that I-IX below are true of (5)

> (5) If it is true that if anyone who is thinking, then that person exists, then, it is true that if I am thinking, then I exist,

[25] Capturing Euclid's well-known proof that the square root of 2 is irrational requires QC with the equality symbol, as the reader can easily verify. I have not studied Descartes' proofs and cannot say whether they are stated in syllogistic logic.

while the negations of I-IX are not. Applying DBP yields that (5) has an epistemic property for *S* only if *S* is able to accept I-IX and reject the negations of I-IX:

I. The three occurrences of "if" are synonymous.

II. The first and third occurrences of "that" are synonymous as propositional prefixes.

III. The second occurrence of "that" is a demonstrative and is not synonymous with the first and third occurrences of "that."

IV. The two occurrences of "true," "it" and "thinking" are synonymous.

V. The three occurrences of "then" are synonymous.

VI. The first occurrence of "if" is paired off with the second occurrence of "then."

VII. The second occurrence of "if" is paired off with the first occurrence of "then."

VIII. The two copula occurrences, "is" and "am," are synonymous despite grammatical differences.

IX. "Exist" and "exists" are synonymous despite grammatical differences.

The reader can easily imagine ways of expanding this list.

The Evil Genius

Recall that Descartes' description at the end of *Meditation I* suggests that the EG has the power and intent to bring about four logically independent states of affairs:

(i) a person accepting what is not true;

(ii) a person rejecting what is true;

(iii) a person being unable to accept what is true;

(iv) a person being unable to reject what is not true.

Three Scenarios

An EG that has the power to impact a person's mental life in such a way as to bring about (i)-(iv), presumably has the power to impact a person's mental life in such a way as to cause a person to experience memory lapses while uttering or thinking (5) to himself,

> (5) If it is true that if anyone who is thinking, then that person exists, then, it is true that if I am thinking, then I exist.

Here are three memory-lapse scenarios under states of affairs (iii) and (iv):

> **Scenario 1**: *S* utters or thinks to himself "If" at the beginning of (5), then continues with "it is true that," but just as he is about to utter or think to himself the "if" after "that," the EG immediately causes *S* to experience a memory lapse so that when *S* utters or thinks to himself the "if" after "that," *S* is unable to remember that this "if" is synonymous with "If" at the beginning of (5) he already

> uttered or thought to himself and as a result *S* is unable (iii) to accept what is true of (5), that the two occurrences of "if" are synonymous; and (iv) reject what is not true of (5), that the two occurrences of "if" are not synonymous.
>
> **Scenario 2**: *S* utters or thinks to himself "that" after "true" in (5), then continues with "if anyone who is thinking, then" but just as he is about to utter or think to himself "that" after "then," the EG, who has been eavesdropping, immediately causes *S* to experience a memory lapse so that when *S* utters or thinks to himself the "that" after "then," *S* is unable to remember that the "that" after "true" is a propositional prefix while the "that" after "then" is a demonstrative, and as a result *S* is unable to (iii) accept what is true of (5), that the two occurrences of "that" are not synonymous; and (iv) reject what is not true of (5), that the two occurrences of "that" are synonymous.
>
> **Scenario 3**: *S* utters or thinks to himself the first occurrences of "am," "thinking," "then," and "exist" but then the EG, who has been eavesdropping, immediately causes *S* to experience memory lapses such that when *S* comes to the point of uttering or thinking to himself the second occurrences of these terms in (5), *S* is unable to remember that they are synonymous with their first occurrences and as a result *S* is unable to (iii) accept what is true of (5), that the two occurrences of each of these terms are synonymous; and (iv) reject what is not true of (5), that the two occurrences of these terms are not synonymous.

Cogito Certainty Has Not Been Achieved

Inspection shows that the doxastic burden principle DBP together with inability to accept what is true and reject what is not true of (5)

> (5) If it is true that if anyone who is thinking, then that person exists, then, it is true that if I am thinking, then I exist,

imply that (6) is false,

> (6) It is certain (for me) that if it is true that if anyone who is thinking, then that person exists, then, it is true that if I am thinking, then I exist.

In Conclusion

The short-term memory lapses I have described are not about whether the EG can bring it about that "*S* is nothing as long as *S* thinks that he is something." Descartes is right about that. The scenarios raise an objection to the certainty of the *Discourse Cogito*. Indeed, *cette vérité*, "I am thinking, therefore I exist," formulated as a valid argument and then as a sentence is a tautology and by implication a necessary truth; so, of course "the most extravagant suppositions of the skeptics could not shake it" for a reason already noted: Epistemic change does not entail semantic change. This does not mean, however, that skeptics cannot shake the *ferme et assurée* of *cette vérité*. They can, needing to be no more extravagant than Descartes' own Evil Genius, aided and abetted by a principle derived from Descartes' principle of clear and distinct perceptions. Without certainty, the *Discourse Cogito* is a failed response to skepticism.

Objections and Replies

Objection 1: Attributing truth and justification to arguments rather than their components is at relatively minor error. Descartes' readers would not have been confused about what he meant. In any case, he can live with a bit of unclarity given that the alternative is abandoning the *Cogito* and with it a response to skepticism. Your analysis is a case of "the cure is worse than the disease."

Reply: The doxastic burden principle DBP together with the EG's power to bring about memory lapses are sufficient to defeat the certainty of the *Discourse Cogito* in its original formulation, "I am thinking, therefore I exist."

Thus, consider this scenario: *S* utters or thinks to himself "I am" at the beginning of the *Cogito* sentence, then utters "thinking," then utters "therefore" and at that very instant, the EG, immediately causes *S* to be unable to remember as he utters or thinks to himself the "I" in "I exist" that it is co-referential with the "I" in "I am" and as a result *S* is unable (iii) to accept what is true of "I am thinking, therefore I exist," that the two occurrences of "I" <u>are</u> co-referential; and (iv) to reject what is not true of "I am thinking, therefore I exist," that the two occurrences of "I" <u>are not</u> co-referential. It follows that Descartes' "first principle of the philosophy I was seeking" is not certain, which dooms his response to skepticism. Needless to add, Descartes scholars would have regarded such a brief refutation of the *Discourse Cogito*'s certainty as giving the matter, and Descartes himself by implication, short shrift and probably would have dismissed it as frivolous. They are less likely to do that after working through the technical details above, I hope.

Objection 2: Your critique may well be just a "one-off." Being the earliest (1637), perhaps the *Discourse Cogito* was in a sense superseded by later versions in *Meditations* (1641) and *Principles* (Latin, 1644; French 1647). Does your critique apply to those versions?

Reply: A thorough analytical comparison of the five *Cogito* versions[26] in light of results in this article would require book-length treatment. For now, I'd like to present textual evidence that *Cogito* versions in *Discourse* and *Principles* stand or fall together.

- The Latin original (AT VIII, 7) of *Principles* uses the epistemic terms terms "*hæc cognitio*" ("this knowledge") and "*est omnium prima & certissima*" ("is the first and most certain") in reference to a *Cogito* also stated in argument form: "*Ac proinde hæc cognitio,* ego cogito, ergo sum*, est omnium & certissima, quæ cuilibet ordine philosophanti occurrat.*" Knowledge implies truth, so …
- The French translation[27] (*Œuvres Complètes*, Kindle Edition, 696) is even closer to the *Discourse* version, using the terms "*cette conclusion*" ("this inference,")

[26] One in French (*Discourse*); one and Latin and one in French (*Meditations*; the French translation by de Luynes appeared in 1647); and one in Latin and one in French (*Principles*.)

[27] Descartes' friend Claude Picot translated *Principles* into French. See Gaukroger 1995, 138 and 386-7. In his preface to the French translation, titled "Author's letter to the translator," Descartes refers to it as "*nette et accomplie*," rendered variously as "elegant and finished"

"*vraie*" ("true,") and "*la plus certaine*" ("the most certain") in reference to the *Cogito*:[28] *… nous ne saurions nous empêcher de croire que cette conclusion: Je pense, donc je suis, ne soit vraie, et par conséquent la première et la plus certaine qui se présente à celui quit conduit ses pensées par ordre.*

- A little later, the Latin original (AT VIII, 7) presents an inference formally similar to the *Cogito*, that proceeds from "seeing and walking" to the familiar ending "*ergo sum*" ("therefore I exist,") followed by a comment that uses an epistemic term "*conclusio non est absolute certa*" ("the conclusion is not absolutely certain"): *Nam si dicam, ego video, vel ego ambulo, ergo sum; & hoc intelligam de visione, aut ambulatione, quæ corpore peragitur, conclusio non est absolute certa.*

- The French translation (*Œuvres Complètes*, Kindle Edition, 698) also implies that inference was involved in the *Cogito*. Verb ("*j'infère*") ("I infer") and noun ("*conclusion*") ("inference") forms are used, as is the epistemic term "*infaillible*" ("certain," "infallible"): *Car si je dis que je vois ou que je marche, et que j'infère de là que je suis; si j'entends parler de l'action qui se fait avec mes yeux ou avec mes jambs, cette conclusion n'est pas tellement infaillible.*

Objection 3: What about the *Cogito* version in *Meditation II*?
Reply: This version reads (Cress 1979, 17) "… the statement 'I am, I exist' is necessarily true every time it is uttered by me or conceived in my mind." The doxastic burden principle DBP and the EG can jointly defeat the *Meditations Cogito* only if this version can paraphrased into a sentence in epistemic form, by (a) replacing "necessarily true" with "certain" and (b) having "it is certain that" prefix the conditional "if I utter or conceive the statement 'I am, I exist,' then the statement 'I am, I exist' is true." However, it is beyond the scope of this paper to determine whether the *Meditations Cogito* text can support (a) and (b), or whether (a) and (b) are consistent with Descartes' other views.
Objection 4: You have shown only that Descartes failed to achieve certainty if his *Discourse Cogito* is interpreted as an argument; not that this *Cogito* must be interpreted as an argument. Thus, you have not ruled out interpretations of this *Cogito* that construe it as something other than an argument. Katz 1986 develops an interpretation according to which the *Cogito* (generically) is an example of "linguistic entailment"; while Sarkar 2003 thinks the *Cogito* is an "experiment."
Reply: Katz and Sarkar are book-length studies of approximately 200 and 300 pages, respectively, a luxury I do not have here. Accordingly, I have deliberately avoided sparring with competing views, concentrating instead on developing my own views and leaving polemical considerations for another time. The Katz and Sarkar interpretations seem to me mistaken but that is not something I can explain in an article of this scope. Briefly, however, we are asked to believe that Descartes had in mind "linguistic entailment" or "an experiment" when he said that the

(Veitch 1912, v), "polished and well-finished" (Haldane 1970, 203), and "polished and thorough" (Cottingham 1985, 179). Picot published two more translations after Descartes' death, in 1651 and 1657. I have not studied them.
[28] Haldane notes (1970, 202) that "the French version frequently differs considerably from the Latin." For details on differences between the two versions, see the AT IXb.

Cogito was "the first principle of the philosophy I was seeking," which quite frankly seems incredible.

Objection 5: For Descartes, "certainty" has an epistemic sense, which rules out grounds for doubt, as well as a metaphysical sense, which rules out the very possibility of such grounds. How does metaphysical certainty affect your arguments?

Reply: It has no effect. The argument used to derive the doxastic burden principle DBP from an epistemic version of Descartes' principle of clear and distinct perceptions (CDP) can also be used to derive a metaphysical certainty version of DBP, MDBP, from a metaphysical certainty version of CDP. It is a topic for another paper to decide whether the EG and MDBT together can also defeat the metaphysical certainty of *Discourse Cogito* sentences in SC and QC.

Objection 6: Descartes offers an alternative to deduction – intuition -- in Rule Three of *Rules for Guiding One's Intelligence*, which he explains as (Clarke 1985, 14)

> "… the conception of a clear and attentive mind, which is so easy and so distinct that there can be no room for doubt about what we are understanding. … Thus everyone can mentally intuit that he exists, that he is thinking, that a triangle is bounded by just three lines, and a sphere by a single surface, and the like."

What about this?

Reply: Descartes abandoned writing *Rules* in 1628, having completed eighteen rules that have a summary followed by text; and three rules that only have a summary. This material was published in 1701, 51 years after Descartes died.

- Because Descartes left *Rules* unfinished, its status as a text that potentially overrides what he says about thinking and existence in works he did finish, such as *Discourse*, *Meditations* and *Principles*, is hardly definitive.
- None of the *Cogito* versions in *Discourse*, *Meditations* and *Principles* mention intuition, suggesting Descartes had abandoned his earlier view.
- Why bother with a *Cogito* in argument form if "everyone can mentally intuit that he exists, that he is thinking"?
- The science-fiction film *The Matrix* shows virtual beings whose "clear and attentive minds" are unaware they are in a computer-generated world.

Objection 7: There is a sense of "certain" in which someone might say "I am certain that the Theory of Relativity is true even though I don't understand what an inertial frame of reference is," basing certainty on the opinion of a professional physicist. So, your doxastic burden principle DBP is false.

Reply: "No, you're not certain that the Theory of Relativity is true. You're just taking someone else's word for it." In any case, this is not an objection Descartes could raise. The concept of certainty he claims for the *Discourse Cogito* is not the hearsay concept. "I am certain that the *Cogito* is true because someone else thinks so" is absurd.

Objection 8: Your doxastic burden principle DBP potentially rules out certainty for any sentence whatever. It is easy to compile lists of items comparable to those

in your scenarios, go through each item, and show how the Evil Genius can sow confusion.

Reply: The scenarios I presented can indeed be modified to apply to a greater range of situations, significantly increasing the reach of the skeptical conclusion. For example, as someone utters or thinks to himself the last five letters of a ten-letter word, the EG interferes and causes that person to be unable to remember the first five letters of that word; or as someone types the first five letters of a word, the EG interferes and causes the person to be unable to remember the last five letters of the word; and so on. Loss of certainty would follow as above.

Descartes himself opened the door to radical skepticism, to which I do not have an answer at the moment. Not everyone, however, walked through the door. Back in my day as a graduate student at Brown University, whenever a member of Roderick Chisholm's epistemology seminar seemed about to raise a skeptical issue, Chisholm would cut him off and ask, "Are you a skeptic?" If the answer was "yes," he'd tell the person he didn't want to discuss skepticism and would simply move on.[29]

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[29] My teachers at the City College of New York, Michael Levin, Arthur Collins and Charles Evans, were not sympathetic to Cartesian dualism. That changed in graduate school at Brown University, where I studied with Roderick Chisholm, Ernest Sosa and James Van Cleve. A youtube lecture on the *Cogito* by Sosa led to this paper. Fred Adams, Roger Ariew, Geoff Giorgi, Gary Rosenkrantz, and Ernest Sosa supplied helpful comments early on. My former colleague at the University of Rhode Island, John Peterson, was the most helpful of all, providing valuable criticisms on the paper's many drafts that led to significant improvements. Thanks to all. I'd also like to thank my former colleague at Northern Illinois University, Don Cress, for providing such clear and readable translations of *Discourse* and *Meditations*.

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